Ohio | Department of Education

# Ohio's State Tests

## ITEM RELEASE

## SPRING 2022

## GEOMETRY

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# Geometry
# Spring 2022 Item Release
# Content Summary and Answer Key

| Question No.* | Item Type | Content Cluster | Content Standard | Depth of Knowledge | Answer Key | Points |
|---|---|---|---|---|---|---|
| 6 | Equation Item | Understand the relationships between lengths, areas, and volumes. | Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures. *(G.GMD.5)* | Level 2 | --- | 1 point |
| 7 | Inline Choice Item | Understand and apply theorems about circles. | Prove that all circles are similar using transformational arguments. *(G.C.1)* | Level 3 | --- | 1 point |
| 8 | Equation Item | Define trigonometric ratios, and solve problems involving right triangles. | Solve problems involving right triangles.★<br>*a*. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. *(G.SRT.8)* | Level 2 | --- | 1 point |
| 10 | Equation Item | Find arc lengths and areas of sectors of circles. | Find arc lengths and areas of sectors of circles.<br>*b*. Derive the formula for the area of a sector, and use it to solve problems. *(G.C.5)* | Level 3 | --- | 1 point |
| 18 | Inline Choice Item | Understand indepen-dence and conditional probability, and use them to interpret data. | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.★ (S.CP.5)* | Level 2 | --- | 1 point |
| 20 | Equation Item | Explain volume formulas, and use them to solve problems. | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ *(G.GMD.3)* | Level 1 | --- | 1 point |

** The question number matches the item number in the Item Level Report in the Centralized Reporting System. The items are numbered sequentially in the practice site.*

# Geometry
# Spring 2022 Item Release
# Content Summary and Answer Key

| Question No.* | Item Type | Content Cluster | Content Standard | Depth of Knowledge | Answer Key | Points |
|---|---|---|---|---|---|---|
| 22 | Multiple Choice Item | Visualize relationships between two-dimensional and three-dimensional objects. | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. *(G.GMD.4)* | Level 2 | A | 1 point |
| 23 | Equation Item | Apply geometric concepts in modeling situations. | Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.★ *(G.MG.2)* | Level 2 | --- | 1 point |
| 26 | Equation Item | Understand indepen-dence and conditional probability, and use them to interpret data. | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★ *(S.CP.4)* | Level 2 | --- | 1 point |

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# Geometry
# Spring 2022 Item Release
# Content Summary and Answer Key

| Question No.* | Item Type | Content Cluster | Content Standard | Depth of Knowledge | Answer Key | Points |
|---|---|---|---|---|---|---|
| 27 | Equation Item | Prove and apply theorems both formally and informally involving similarity using a variety of methods. | Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles. *(G.SRT.5)* | Level 2 | --- | 1 point |
| 28 | Multiple Choice Item | Understand similarity in terms of similarity transform-ations. | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. *(G.SRT.2)* | Level 1 | D | 1 point |
| 30 | Multiple Choice Item | Explain volume formulas, and use them to solve problems. | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ *(G.GMD.3)* | Level 2 | B | 1 point |
| 31 | Equation Item | Use the rules of probability to compute probabilities of compound events in a uniform probability model. | Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★ *(S.CP.7)* | Level 2 | --- | 1 point |

** The question number matches the item number in the Item Level Report in the Centralized Reporting System. The items are numbered sequentially in the practice site.*

# Geometry
# Spring 2022 Item Release
# Content Summary and Answer Key

| Question No.* | Item Type | Content Cluster | Content Standard | Depth of Knowledge | Answer Key | Points |
|---|---|---|---|---|---|---|
| 32 | Equation Item | Understand similarity in terms of similarity transform-ations. | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. *(G.SRT.2)* | Level 2 | --- | 2 points |
| 34 | Multiple Choice Item | Define trigonometric ratios, and solve problems involving right triangles. | Explain and use the relationship between the sine and cosine of complementary angles. *(G.SRT.7)* | Level 1 | D | 1 point |
| 37 | Inline Choice Item | Prove and apply theorems both formally and informally involving similarity using a variety of methods. | Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles. *(G.SRT.5)* | Level 3 | --- | 1 point |
| 38 | Multiple Choice Item | Understand indepen-dence and conditional probability, and use them to interpret data. | Understand the conditional probability of A given B as $\frac{P(A \text{ and } B)}{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★ *(S.CP.3)* | Level 1 | A | 1 point |

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# Geometry
# Spring 2022 Item Release
# Content Summary and Answer Key

| Question No.* | Item Type | Content Cluster | Content Standard | Depth of Knowledge | Answer Key | Points |
|---|---|---|---|---|---|---|
| 42 | Equation Item | Apply geometric concepts in modeling situations. | Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★ *(G.MG.3)* | Level 2 | --- | 1 point |
| 43 | Gap Match Item | Prove geometric theorems both formally and informally using a variety of methods. | Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)* | Level 3 | --- | 1 point |
| 45 | Multi-Select Item | Experiment with transform-ations in the plane. | Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself.<br>*a.* Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. *(G.CO.3)* | Level 1 | A, C, D, F | 1 point |
| 48 | Multiple Choice Item | Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements. | Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle. (G.GPE.4)* | Level 2 | D | 1 point |

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# Depth of Knowledge (DOK)

DOK refers to the complexity of thinking required to complete a task in a given item. Items with a DOK 1 designation focus on the recall of information, such as definitions and terms, and simple procedures. Items with a DOK 2 designation require students to make decisions, solve routine problems, perform calculations, or recognize patterns. Items with a DOK 3 designation feature higher-order cognitive tasks. These DOK 3 tasks include but are not limited to: critiquing a statement and forming a conclusion; explaining, justifying, or proving a statement; or approaching abstract, complex, open-ended, and non-routine problems. Each grade's blueprint contains information about the number of points of opportunity students will encounter at each DOK level.

**Table 1: *Math Descriptors* – Applying Depth of Knowledge Levels for Mathematics (Webb, 2002) & NAEP 2002 Mathematics Levels of Complexity**
(M. Petit, Center for Assessment 2003, K. Hess, Center for Assessment, updated 2006)

| Level 1<br>Recall | Level 2<br>Skills/Concepts | Level 3<br>Strategic Thinking | Level 4*<br>Extended Thinking |
|---|---|---|---|
| a. Recall, observe, or recognize a fact, definition, term, or property<br>b. Apply/compute a well-known algorithm (e.g., sum, quotient)<br>c. Apply a formula<br>d. Determine the area or perimeter of rectangles or triangles given a drawing and labels<br>e. Identify a plane or three-dimensional figure<br>f. Measure<br>g. Perform a specified or routine procedure (e.g., apply rules for rounding)<br>h. Evaluate an expression<br>i. Solve a one-step word problem<br>j. Retrieve information from a table or graph | a. Classify plane and three-dimensional figures<br>b. Interpret information from a simple graph<br>c. Use models to represent mathematical concepts<br>d. Solve a routine problem requiring multiple steps/ decision points, or the application of multiple concepts<br>e. Compare and/or contrast figures or statements<br>f. Construct 2-dimensional patterns for 3-dimensional models, such as cylinders and cones<br>g. Provide justifications for steps in a solution process<br>h. Extend a pattern | a. Interpret information from a complex graph<br>b. Explain thinking when more than one response is possible<br>c. Make and/or justify conjectures<br>d. Use evidence to develop logical arguments for a concept<br>e. Use concepts to solve non-routine problems<br>f. Perform a procedure with multiple steps and multiple decision points<br>g. Generalize a pattern<br>h. Describe, compare, and contrast solution methods<br>i. Formulate a mathematical model for a complex situation<br>j. Provide mathematical justifications | a. Relate mathematical concepts to other content areas<br>b. Relate mathematical concepts to real-world applications in new situations<br>c. Apply a mathematical model to illuminate a problem, situation<br>d. Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results<br>e. Design a mathematical model to inform and solve a practical or abstract situation<br>f. Develop generalizations of the results obtained and the strategies used and apply them to new problem situations |

**Table 1** *continued on next page.*

| Level 1<br>Recall | Level 2<br>Skills/Concepts | Level 3<br>Strategic Thinking | Level 4*<br>Extended Thinking |
|---|---|---|---|
| k. Recall, identify, or make conversions between and among representations or numbers (fractions, decimals, and percents), or within and between customary and metric measures<br>l. Locate numbers on a number line, or points on a coordinate grid<br>m. Solve linear equations<br>n. Represent math relationships in words, pictures, or symbols<br>o. Read, write, and compare decimals in scientific notation | i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps<br>j. Translate between tables, graphs, words and symbolic notation<br>k. Make direct translations between problem situations and symbolic notation<br>l. Select a procedure according to criteria and perform it<br>m. Specify and explain relationships between facts, terms, properties, or operations<br>n. Compare, classify, organize, estimate, or order data | k. Solve a multiple-step problem and provide support with a mathematical explanation that justifies the answer<br>l. Solve 2-step linear equations/ inequalities in one variable over the rational numbers, interpret solution(s) in the original context, and verify reasonableness of results<br>m. Translate between a problem situation and symbolic notation that is not a direct translation<br>n. Formulate an original problem, given a situation<br>o. Analyze the similarities and differences between procedures<br>p. Draw conclusion from observations or data, citing evidence | g. Apply one approach among many to solve problems<br>h. Apply understanding in a novel way, providing an argument/ justification for the application<br><br>***NOTE:*** *Level 4 involves such things as complex restructuring of data or establishing and evaluating criteria to solve problems.* |

***Note: Ohio's State Tests only assess and measure DOK Levels 1 – 3 in grades K – 12. Level 4 is included in this table for informational purposes only.***

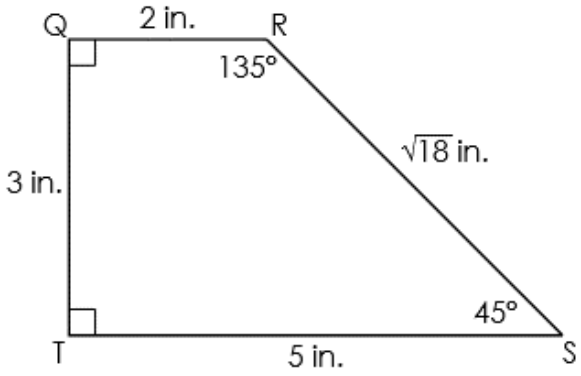
# Geometry
# Spring 2022 Item Release

# Question 6

# Question and Scoring Guidelines

# Question 6

Quadrilateral QRST is shown, with side lengths in inches (in.) and angle measurements in degrees.




Lynn draws quadrilateral WXYZ, which is similar to quadrilateral QRST, with $WX = 5$ in.

Enter values in the blank boxes to make the statement true.

The length of $\overline{WZ}$ is [ ] inches and the measure of angle Y is [ ] degrees.

**Points Possible:** 1

**Content Cluster:** Understand the relationships between lengths, areas, and volumes.

**Content Standard:** Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures. *(G.GMD.5)*

**Depth of Knowledge:** Level 2
c. Use models to represent mathematical concepts
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

- The length of $\overline{WZ}$ is 7.5 inches and the measure of angle Y is 45 degrees.

Other Correct Responses

- any equivalent values

For full credit (1 point), the student's response satisfies the bullet below.

- The student enters 7.5 inches as the side length and 45 degrees as the angle length.

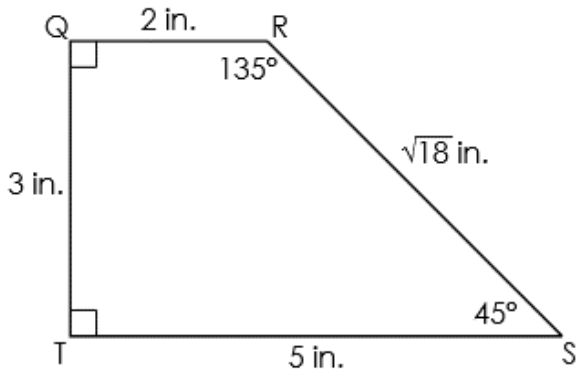
# Geometry
# Spring 2022 Item Release

# Question 6

# Sample Responses

## Sample Response: 1 point

Quadrilateral QRST is shown, with side lengths in inches (in.) and angle measurements in degrees.




Lynn draws quadrilateral WXYZ, which is similar to quadrilateral QRST, with $WX = 5$ in.

Enter values in the blank boxes to make the statement true.

The length of $\overline{WZ}$ is 7.5 inches and the measure of angle Y is 45 degrees.

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correctly completed sentence providing evidence of understanding that in similar figures scaling of one side results in scaling of all other sides with the same constant scale factor, but this scaling does not affect angle measures. Therefore, corresponding angles of similar figures are congruent.

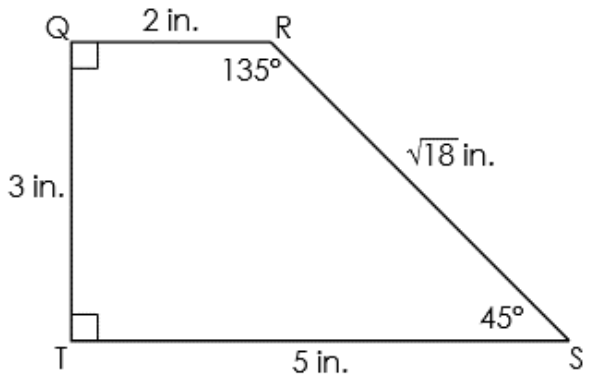
In the two similar figures, sides QR = 2 and WX = 5 are corresponding. The scale factor is $\frac{\mathrm{WX}}{\mathrm{QR}} = \frac{5}{2}$ = 2.5 and $\frac{\mathrm{WZ}}{\mathrm{QT}} = 2.5$. Therefore, WZ = 2.5 · QT or WZ = 2.5 · 3 = 7.5.

Since the measure of corresponding angles of similar figures are equal and $m\angle S = 45°$, then $m\angle \mathrm{Y} = 45°$ too.

The complete sentence is, "The length of WZ is 7.5 inches and the measure of angle Y is 45 degrees."

## Sample Response: 1 point

Quadrilateral QRST is shown, with side lengths in inches (in.) and angle measurements in degrees.




Lynn draws quadrilateral WXYZ, which is similar to quadrilateral QRST, with $WX = 5$ in.

Enter values in the blank boxes to make the statement true.

The length of $\overline{WZ}$ is $\frac{15}{2}$ inches and the measure of angle Y is 45 degrees.

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correctly completed sentence providing evidence of understanding that in similar figures scaling of one side results in scaling of all other sides with the same constant scale factor, but this scaling does not affect angle measures. Therefore, corresponding angles of similar figures are congruent.

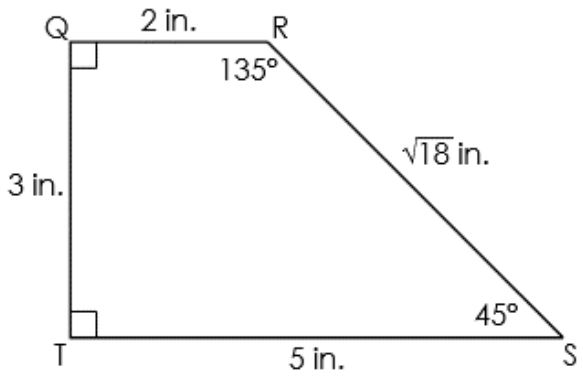
In the two similar figures, sides QR = 2 and WX = 5 are corresponding. The scale factor is $\frac{\mathrm{WX}}{\mathrm{QR}} = \frac{5}{2}$ and $\frac{\mathrm{WZ}}{\mathrm{QT}} = \frac{5}{2}$. Therefore, $\frac{\mathrm{WZ}}{\mathrm{QT}} = \frac{\mathrm{WZ}}{3} = \frac{5}{2}$ or $\mathrm{WZ} = \frac{5}{2} \cdot 3 = \frac{15}{2}$.

Since the measure of corresponding angles of similar figures are equal and m∠S = 45°, then m∠Y = 45° too.

The complete sentence is, "The length of WZ is $\frac{15}{2}$ inches and the measure of angle Y is 45 degrees."

## Sample Response: 0 points

Quadrilateral QRST is shown, with side lengths in inches (in.) and angle measurements in degrees.




Lynn draws quadrilateral WXYZ, which is similar to quadrilateral QRST, with $WX = 5$ in.

Enter values in the blank boxes to make the statement true.

The length of $\overline{WZ}$ is 6 inches and the measure of angle Y is 50 degrees.

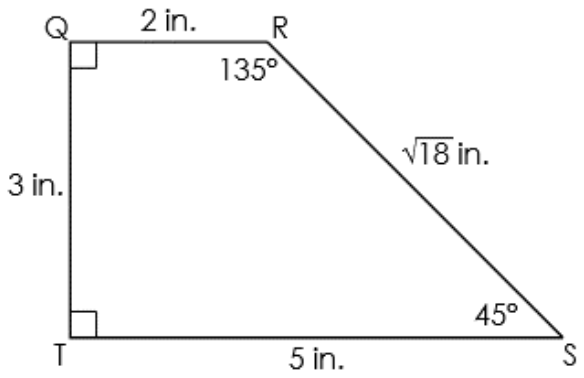
**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrectly completed sentence.

The student may think the rule for the side lengths is to add 3 instead of setting up a ratio of 2:5. The student may then add 5 to the angle measure, or 45 + 5 = 50, because the length of WX is 5 inches.

## Sample Response: 0 points

Quadrilateral QRST is shown, with side lengths in inches (in.) and angle measurements in degrees.




Lynn draws quadrilateral WXYZ, which is similar to quadrilateral QRST, with $WX = 5$ in.

Enter values in the blank boxes to make the statement true.

The length of $\overline{WZ}$ is 3 inches and the measure of angle Y is 5 degrees.

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrectly completed sentence.

The student may think that corresponding sides of similar figures must be the same, so the length of segment WZ is equal to the length of segment QT. The student may then confuse the given side length, 5 in, with the measure of the angle Y in the similar quadrilateral.

# Geometry
# Spring 2022 Item Release

# Question 7

# Question and Scoring Guidelines

# Question 7

Circle M has radius MN, and Circle P has radius PQ. Points M and P are distinct points.

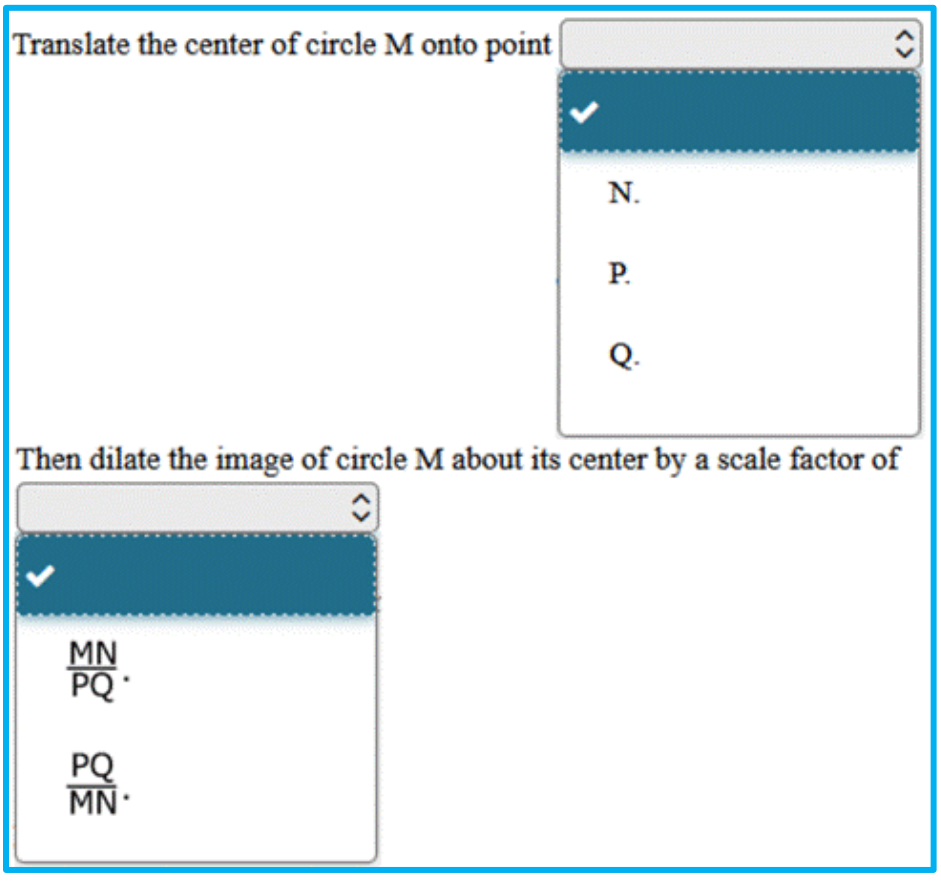
Select a term for each blank box to complete the statements describing how to prove that the circles are similar.

Translate the center of circle M onto point [ ]

Then dilate the image of circle M about its center by a scale factor of [ ]

Because this sequence of similarity transformations maps one circle onto the other, the two circles are similar.

**Drop Down Choices:**



**Points Possible:** 1

**Content Cluster:** Understand and apply theorems about circles.

**Content Standard:** Prove that all circles are similar using transformational arguments. *(G.C.1)*

**Depth of Knowledge:** Level 3
d. Use evidence to develop logical arguments for a concept
e. Use concepts to solve non-routine problems

# Scoring Guidelines

Exemplar Response

- Translate the center of circle M onto point P.
- Then dilate the image of circle M about its center by a scale factor of $\frac{PQ}{MN}$.
- Because this sequence of similarity transformations maps one circle onto the other, the two circles are similar.

Other Correct Responses

- N/A

For full credit (1 point), the student's response satisfies the bullet below.

- The student completes the statement explaining how to prove circles are similar.

# Geometry
# Spring 2022 Item Release

# Question 7

# Sample Responses

## Sample Response: 1 point

Circle M has radius MN, and Circle P has radius PQ. Points M and P are distinct points.

Select a term for each blank box to complete the statements describing how to prove that the circles are similar.

Translate the center of circle M onto point P.

Then dilate the image of circle M about its center by a scale factor of $\frac{PQ}{MN}$.

Because this sequence of similarity transformations maps one circle onto the other, the two circles are similar.

**Notes on Scoring**

This response earns full credit (1 point) because it shows two correctly completed sentences to prove a similarity of two circles using similarity transformations.

To prove that circle M and circle P are similar, show that circle M can overlay circle P by applying a sequence of transformations—a translation and a dilation by the scale factor *k* about the center of the translated image of circle M.

To get the centers of the circles to coincide, translate the center of circle M onto the center of circle P. The first complete sentence is, "Translate the center of circle M onto point P."

To get the circles to completely overlay, dilate the translated image of circle M by a scale factor $k = \frac{radius\ of\ the\ image}{radius\ of\ the\ preimage}$ or $\frac{radius\ of\ circle P}{radius\ of\ circle\ M}$ or $\frac{PQ}{MN}$. The second complete sentence is, "Then dilate the image of circle M about its center by a scale factor of $\frac{PQ}{MN}$."

## Sample Response: 0 points

Circle M has radius MN, and Circle P has radius PQ. Points M and P are distinct points.

Select a term for each blank box to complete the statements describing how to prove that the circles are similar.

Translate the center of circle M onto point P.

Then dilate the image of circle M about its center by a scale factor of $\frac{MN}{PQ}$.

Because this sequence of similarity transformations maps one circle onto the other, the two circles are similar.

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect scale factor.

The student correctly identifies the point where the center of circle M is translated but inverts the scale factor and uses $\frac{MN}{PQ}$ instead of $\frac{PQ}{MN}$.

## Sample Response: 0 points

Circle M has radius MN, and Circle P has radius PQ. Points M and P are distinct points.

Select a term for each blank box to complete the statements describing how to prove that the circles are similar.

Translate the center of circle M onto point N.

Then dilate the image of circle M about its center by a scale factor of $\frac{MN}{PQ}$.

Because this sequence of similarity transformations maps one circle onto the other, the two circles are similar.

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect point and an incorrect scale factor.

The student may assume that the center of circle M should be translated onto point N since point N is part of the same circle with radius MN. The student may then invert the scale factor and uses $\frac{MN}{PQ}$ instead of $\frac{PQ}{MN}$.

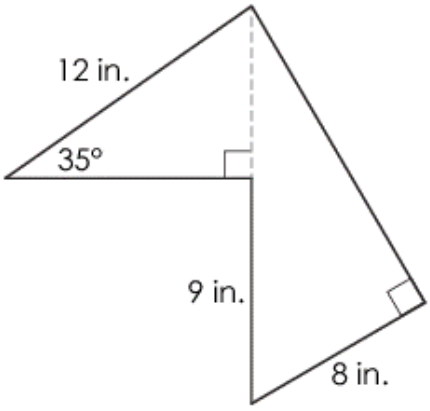
# Geometry
# Spring 2022 Item Release

# Question 8

# Question and Scoring Guidelines

# Question 8

Kevin is creating a mosaic out of tiles. One of the tiles is shown.




What is the perimeter of the tile, rounded to the nearest inch (in.)?

*in.*

**Points Possible:** 1

**Content Cluster:** Define trigonometric ratios, and solve problems involving right triangles.

**Content Standard:** Solve problems involving right triangles.★
*a*. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. *(G.SRT.8)*

**Depth of Knowledge:** Level 2
d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps

# Scoring Guidelines

Exemplar Response

- 53

Other Correct Responses

- 52.5 to 53, inclusive

For this item, a full-credit response includes:

- the correct perimeter (1 point).

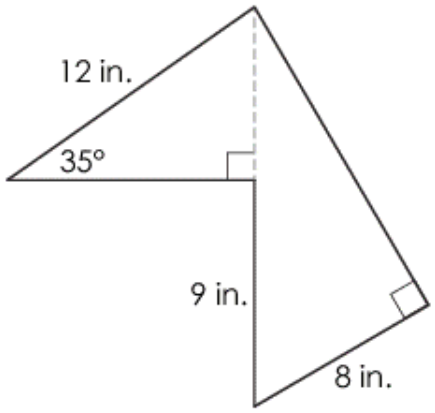
# Geometry
# Spring 2022 Item Release

# Question 8

# Sample Responses

## Sample Response: 1 point

Kevin is creating a mosaic out of tiles. One of the tiles is shown.




What is the perimeter of the tile, rounded to the nearest inch (in.)?

53 *in.*

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct perimeter of the tile.

The shown tile is a composite figure consisting of two right triangles. The perimeter of the tile is the sum of all side lengths of the tile, represented by the solid line segments.

In the triangle on the left, the hypotenuse is 12 and the unknown horizontal side, to be called $x$, is adjacent to the angle of 35°. In right triangles, trigonometric ratios can be used to find the unknown sides. The cosine of an acute angle is the ratio of the length of the adjacent side to the length of hypotenuse. Based on this definition, $\cos 35° = \frac{x}{12}$. After multiplying both sides of this equation by 12, $x = 12 \bullet \cos 35°$, or $x \approx 9.8$, when rounded to the nearest tenth of an inch. The length of the dotted line, to be called $y$, can be found using the definition of sine, or the ratio of the length of the opposite side to the length of hypothenuse as

$$\sin 35° = \frac{y}{12} \qquad y = 12 \bullet \sin 35° \qquad y \approx 6.9$$

This value will be used to find the length of the entire hypothenuse of the triangle on the right as 9 + 6.9 = 15.9 in.

A calculation of the perimeter of the tile should include the length of the unknown side, to be called $z$, of the triangle on the right. To find $z$, use the Pythagorean theorem as

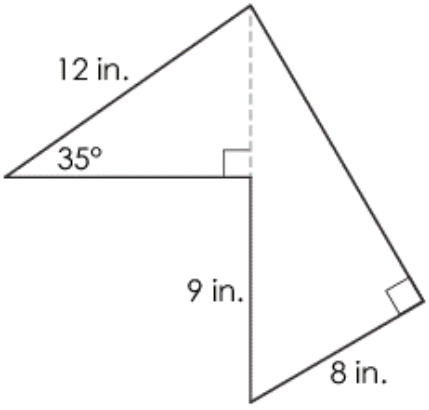
$$15.9^2 = 8^2 + z^2 \qquad 252.81 = 64 + z^2 \qquad 252.81 - 64 = z^2$$

$$188.81 = z^2 \qquad z \approx 13.74$$

The perimeter of the tile is $P$ = 12 + 9.8 + 9 + 8 + 13.74 = 52.54 or 53 in., when rounded to the nearest inch.

## Sample Response: 1 point

Kevin is creating a mosaic out of tiles. One of the tiles is shown.




What is the perimeter of the tile, rounded to the nearest inch (in.)?

52.55 *in.*

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct perimeter of the tile.

The shown tile is a composite figure consisting of two right triangles. The perimeter of the tile is the sum of all side lengths of the tile, represented by the solid line segments.

In the triangle on the left, the hypotenuse is 12 and the unknown horizontal side, to be called $x$, is adjacent to the angle of 35°. In right triangles, trigonometric ratios can be used to find the unknown sides. The cosine of an acute angle is the ratio of the length of the adjacent side to the length of hypotenuse. Based on this definition, $\cos 35° = \frac{x}{12}$. After multiplying both sides of this equation by 12, $x = 12 \cdot \cos 35°$, or $x \approx 9.83$, when rounded to the nearest hundredth of an inch.

The length of the dotted line, to be called $y$, can be found using the definition of sine, or the ratio of the length of the opposite side to the length of hypothenuse as

$$\sin 35° = \frac{y}{12} \qquad y = 12 \cdot \sin 35° \qquad y \approx 6.88$$

This value will be used to find the length of the entire hypothenuse of the triangle on the right as 9 + 6.88 = 15.88 in.

A calculation of the perimeter of the tile should include the length of the unknown side, to be called $z$, of the triangle on the right. To find $z$, use the Pythagorean theorem as

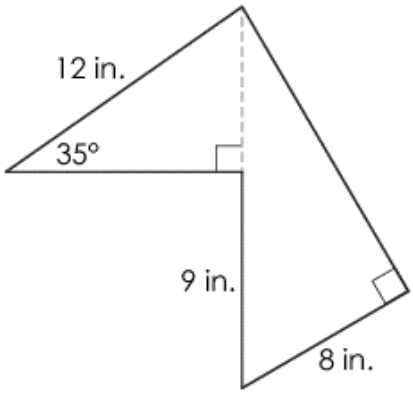
$$15.88^2 = 8^2 + z^2 \qquad 252.1744 = 64 + z^2 \qquad 252.1744 - 64 = z^2$$

$$188.1744 = z^2 \qquad z \approx 13.72$$

The perimeter of the tile is $P = 12 + 9.83 + 9 + 8 + 13.72 = 52.55$ in., when rounded to the nearest hundredths instead of the nearest inch. This result is accepted for the full credit because it shows more precision than was necessary to answer the question.

## Sample Response: 0 points

Kevin is creating a mosaic out of tiles. One of the tiles is shown.




What is the perimeter of the tile, rounded to the nearest inch (in.)?

52 *in.*

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | |

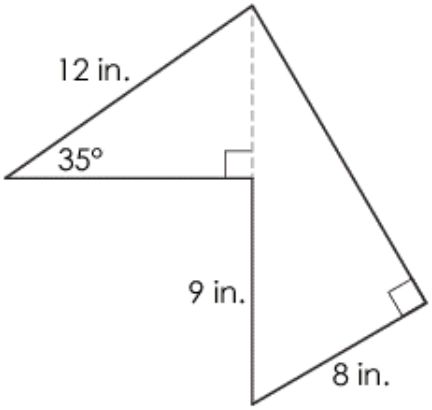
**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect perimeter of the tile.

The student may calculate the perimeter correctly, $P = 12 + 9.83 + 9 + 8 + 13.72 = 52.55$, but may perform a rounding error and round the final answer down to 52.

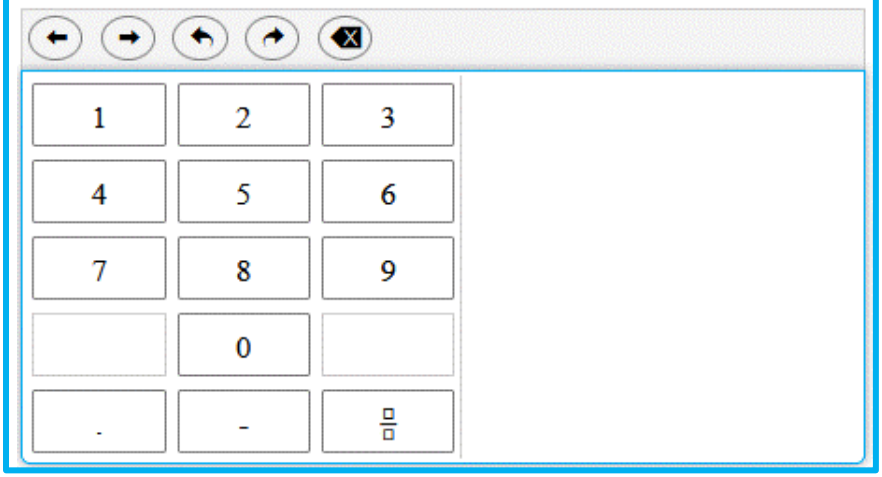
## Sample Response: 0 points

Kevin is creating a mosaic out of tiles. One of the tiles is shown.




What is the perimeter of the tile, rounded to the nearest inch (in.)?

51 *in.*



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect perimeter of the tile.

The student may attempt to complete calculations for the perimeter but may round each of the intermediate calculations for the missing sides down to the nearest inch, then may add the sides together to get a perimeter of 51 inches.

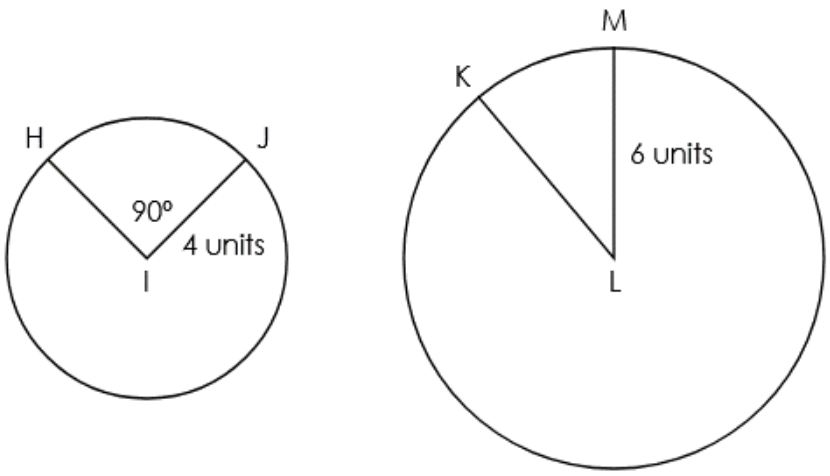
# Geometry
# Spring 2022 Item Release

# Question 10

# Question and Scoring Guidelines

# Question 10

Circles I and L are shown. Sector HIJ and sector KLM have the same area.




What is the measure, in degrees, of angle KLM?

degrees

**Points Possible:** 1

**Content Cluster:** Find arc lengths and areas of sectors of circles.

**Content Standard:** Find arc lengths and areas of sectors of circles.
*b*. Derive the formula for the area of a sector, and use it to solve problems. *(G.C.5)*

**Depth of Knowledge:** Level 3
f. Perform a procedure with multiple steps and multiple decision points
i. Formulate a mathematical model for a complex situation

# Scoring Guidelines

Exemplar Response

- 40 degrees

Other Correct Responses

- any equivalent value

For full credit (1 point), the student's response satisfies the bullet below.

- The student gives an angle measure of 40 degrees.

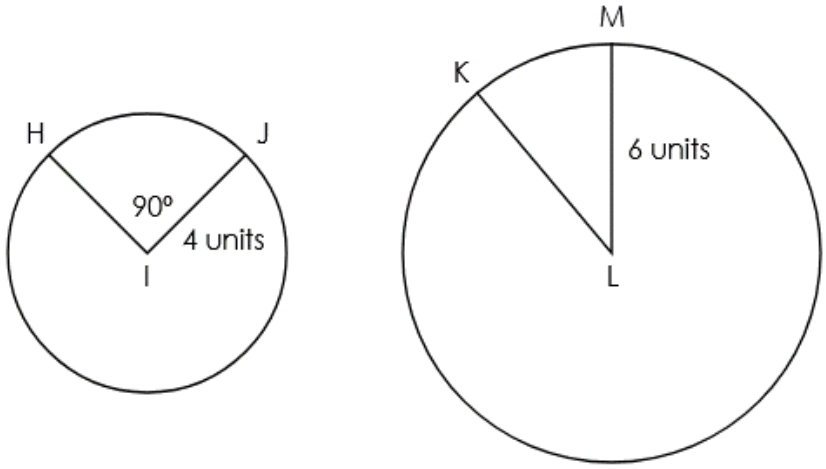
# Geometry
# Spring 2022 Item Release

# Question 10

# Sample Responses

## Sample Response: 1 point

Circles I and L are shown. Sector HIJ and sector KLM have the same area.




What is the measure, in degrees, of angle KLM?

40 *degrees*

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | |

**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct measure of the unknown central angle of the circle.

To calculate the measure of angle KLM, use the formula for the area of the sector A = $\frac{\pi\theta}{360}r^2$ and let the measure of angle KLM be $\theta$.

The area of sector HIJ can be calculated in several ways. One of them is by using a formula $A_{HIJ} = \frac{\pi 90}{360}4^2 = 4\pi$. Another way is by noticing that the area of sector HIJ is a quarter of the area of the left circle because its central angle HIJ = 90° or a quarter of 360°. Therefore, $A_{HIJ} = \frac{1}{4}\pi r^2$ or $A_{HIJ} = \frac{1}{4}\pi = 4\pi$.

The area of sector KLM is $A_{KLM} = \frac{\pi\theta}{360}6^2$. Since the areas of the two sectors are given as equal, set the two expressions equal and solve the equation for the unknown angle $\theta$.

$\frac{\pi 90}{360}4^2 = \frac{\pi\theta}{360}6^2$ Multiply both sides of the equation by $\frac{360}{\pi}$

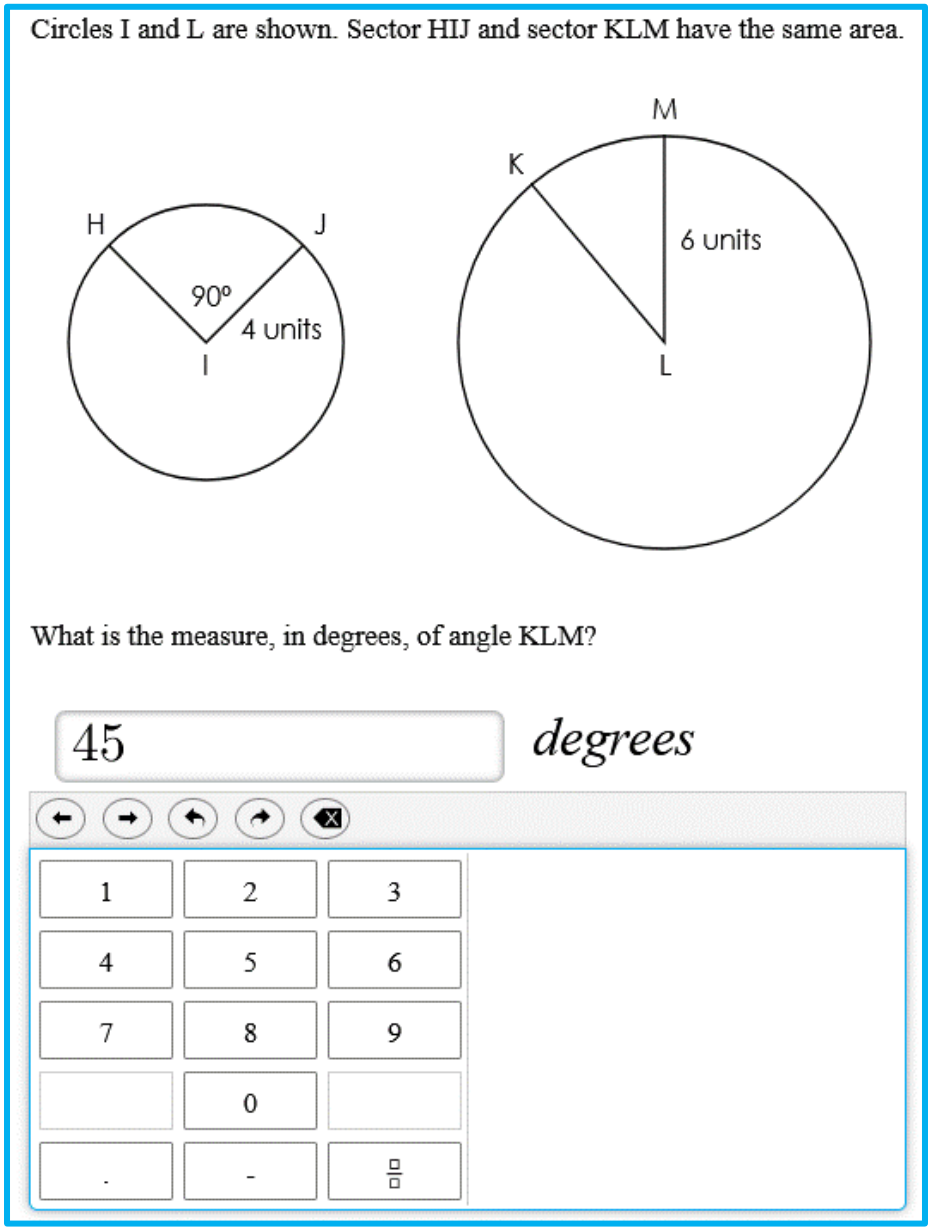
$$90 \cdot 4^2 = \theta \cdot 6^2$$

$$90 \cdot 16 = \theta \cdot 36$$

$\theta = \frac{90 \cdot 16}{36}$ Divide both sides by 36

$$\theta = 40°$$

## Sample Response: 0 points

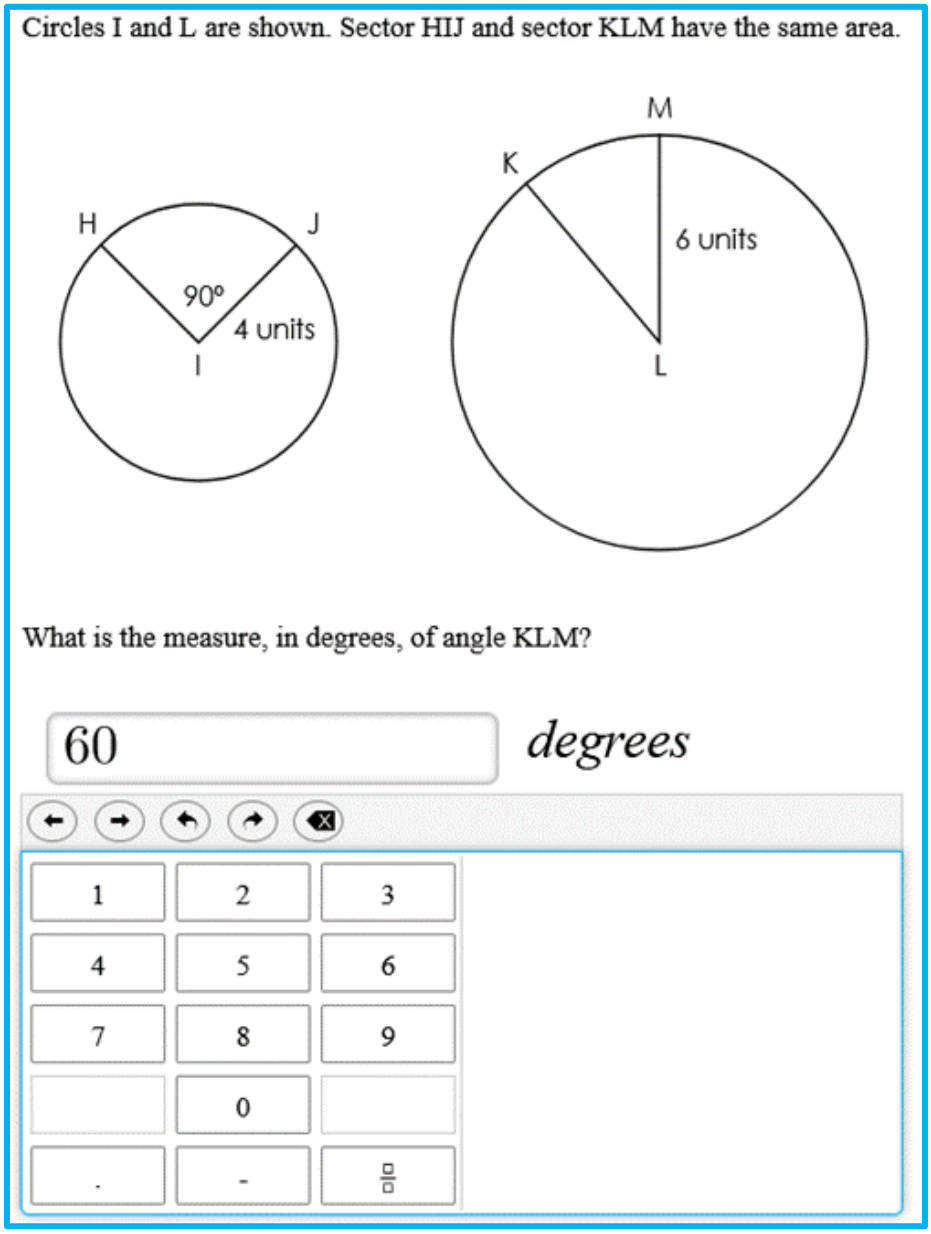



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect measure of the angle KLM.

The student may assume that the measure of this angle is a half of the measure of the angle HIJ because of the visual appearance of the figure.

## Sample Response: 0 points




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect measure of the angle KLM.

The student may assume that the measures of the angles and the lengths of radii relate in the same way as 4 : 6 or 60 : 90.

# Geometry
# Spring 2022 Item Release

# Question 18

# Question and Scoring Guidelines

# Question 18

Greg buys a single coffee every weekday. He determines that the probability that he will buy a coffee on a random day in a 28-day period is $\frac{20}{28}$. Greg also looks at weather records for the 28 days and sees that the temperature was above $24\ ^\circ C$ on 7 of the 28 days.

Greg determines that the probability that the temperature will be above $24\ ^\circ C$ and that he will buy a coffee is $\frac{5}{28}$.

Select a phrase and a word to complete the sentence identifying the effect of the average daily temperature when it was above $24\ ^\circ C$ and Greg purchasing a coffee on a random day in the 28-day period.

If the average daily temperature on a day is 24°*C* or higher, it is [ ] that Greg will buy a coffee because the two events are [ ]

**Drop Down Choices:**

If the average daily temperature on a day is 24°*C* or higher, it is [ ] that Greg will buy a coffee because the two events are [ ]

- less likely
- more likely
- equally likely

- dependent.
- independent.

**Points Possible:** 1

**Content Cluster:** Understand independence and conditional probability, and use them to interpret data.

**Content Standard:** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.★ (S.CP.5)*

**Depth of Knowledge:** Level 2
l. Select a procedure according to criteria and perform it
m. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

- If the average daily temperature on a day is 24°C or higher, it is **equally likely** that Greg will buy a coffee because the two events are **independent**.

Other Correct Responses

- N/A

For full credit (1 point), the student's response satisfies the bullet below.

- The student correctly identifies that it is equally likely that Greg will buy a coffee when the temperature is 24°C or higher and that the events are independent.

# Geometry
# Spring 2022 Item Release

# Question 18

# Sample Responses

## Sample Response: 1 point

Greg buys a single coffee every weekday. He determines that the probability that he will buy a coffee on a random day in a 28-day period is $\frac{20}{28}$. Greg also looks at weather records for the 28 days and sees that the temperature was above $24\ ^\circ C$ on 7 of the 28 days.

Greg determines that the probability that the temperature will be above $24\ ^\circ C$ and that he will buy a coffee is $\frac{5}{28}$.

Select a phrase and a word to complete the sentence identifying the effect of the average daily temperature when it was above $24\ ^\circ C$ and Greg purchasing a coffee on a random day in the 28-day period.

If the average daily temperature on a day is $24°C$ or higher, it is

equally likely ⇕ that Greg will buy a coffee because the two events are

independent. ⇕

### Notes on Scoring

This response earns full credit (1 point) because it correctly compares the probability of the two events and recognizes that the two events are independent.

Two events are independent if the condition for independence of two events, or $P(A \text{ and } B) = P(A) \cdot P(B)$ is met.

To solve the problem, compare the value of P(A and B) and the value of a product, $P(A) \cdot P(B)$, and verify they are equal.

Since $P(A \text{ and } B) = \frac{5}{28}$ and $P(A) \cdot P(B) = \frac{7}{28} \cdot \frac{20}{28} = \frac{5}{28}$, the condition for independence of two events is met, and the events are independent. Since the outcome of an independent event does not affect the outcome of another independent event, and thus, the occurrences of these two events are equally likely.

The complete sentence is "If the average temperature on a day is 24°C or higher, it is equally likely that Greg will buy a coffee because the two events are independent."

## Sample Response: 0 points

Greg buys a single coffee every weekday. He determines that the probability that he will buy a coffee on a random day in a 28-day period is $\frac{20}{28}$. Greg also looks at weather records for the 28 days and sees that the temperature was above $24\ ^\circ C$ on 7 of the 28 days.

Greg determines that the probability that the temperature will be above $24\ ^\circ C$ and that he will buy a coffee is $\frac{5}{28}$.

Select a phrase and a word to complete the sentence identifying the effect of the average daily temperature when it was above $24\ ^\circ C$ and Greg purchasing a coffee on a random day in the 28-day period.

If the average daily temperature on a day is $24°C$ or higher, it is

less likely ⇕ that Greg will buy a coffee because the two events are

dependent. ⇕

**Notes on Scoring**

This response earns no credit (0 points) because it incorrectly compares the probability of the two events and does not recognize that the two events are independent.

The student may not understand that $P(A \text{ and } B) = P(A) \cdot P(B)$ since $P(A \text{ and } B) = \frac{5}{28} = P(A) \cdot P(B) = \frac{7}{28} \cdot \frac{20}{28} = \frac{5}{28}$, so A and B are independent. The student may think that the temperature being higher will make the student less likely to buy coffee.

## Sample Response: 0 points

Greg buys a single coffee every weekday. He determines that the probability that he will buy a coffee on a random day in a 28-day period is $\frac{20}{28}$. Greg also looks at weather records for the 28 days and sees that the temperature was above $24\ ^\circ C$ on 7 of the 28 days.

Greg determines that the probability that the temperature will be above $24\ ^\circ C$ and that he will buy a coffee is $\frac{5}{28}$.

Select a phrase and a word to complete the sentence identifying the effect of the average daily temperature when it was above $24\ ^\circ C$ and Greg purchasing a coffee on a random day in the 28-day period.

If the average daily temperature on a day is $24°C$ or higher, it is

less likely ⇕ that Greg will buy a coffee because the two events are

independent. ⇕

### Notes on Scoring

This response earns no credit (0 points) because it incorrectly compares the probability of the two events even though it recognizes that the two events are independent.

The student correctly recognizes that if $P(A \text{ and } B) = P(A) \cdot P(B)$, or $P(A \text{ and } B) = \frac{5}{28} = P(A) \cdot P(B) = \frac{7}{28} \cdot \frac{20}{28} = \frac{5}{28}$, then *A* and *B* are independent. The student may not realize that the outcome of an independent event does not affect the outcome of another independent event, and thus, the occurrences of these two events are equally likely.

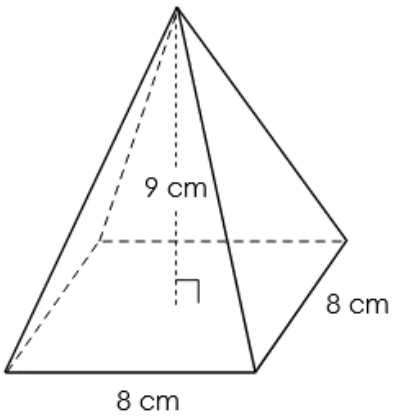
# Geometry
# Spring 2022 Item Release

# Question 20

# Question and Scoring Guidelines

# Question 20

A square pyramid is shown, with dimensions in centimeters (cm).




What is the volume, in cubic centimeters, of the pyramid?

*cubic centimeters*

**Points Possible:** 1

**Content Cluster:** Explain volume formulas, and use them to solve problems.

**Content Standard:** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ *(G.GMD.3)*

**Depth of Knowledge:** Level 1
c. Apply a formula
j. Retrieve information from a table or graph

# Scoring Guidelines

Exemplar Response

- 192 cubic centimeters

Other Correct Responses

- any equivalent value

For full credit (1 point), the student's response satisfies the bullet below.

- The student determines the volume is 192 cubic centimeters.

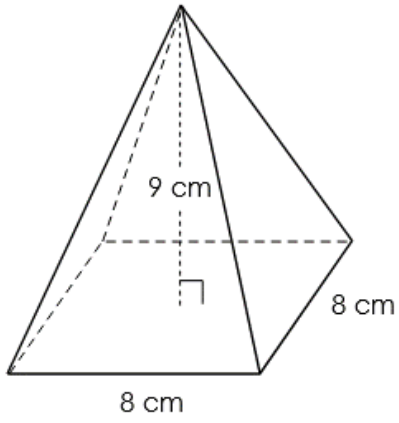
# Geometry
# Spring 2022 Item Release

# Question 20

# Sample Responses

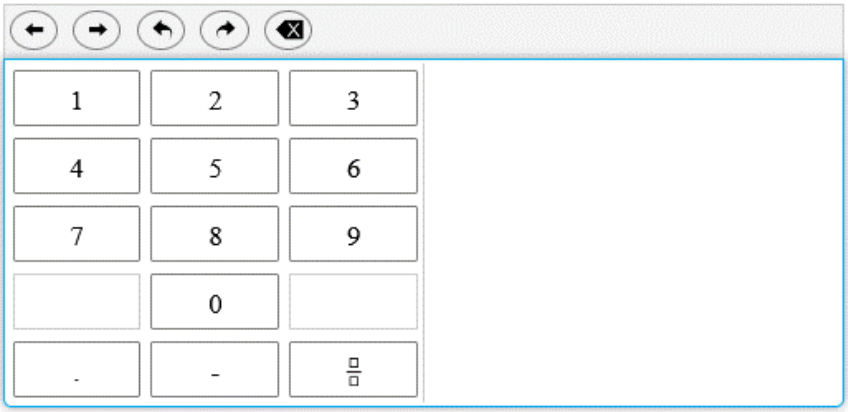
## Sample Response: 1 point

A square pyramid is shown, with dimensions in centimeters (cm).




What is the volume, in cubic centimeters, of the pyramid?

192 *cubic centimeters*



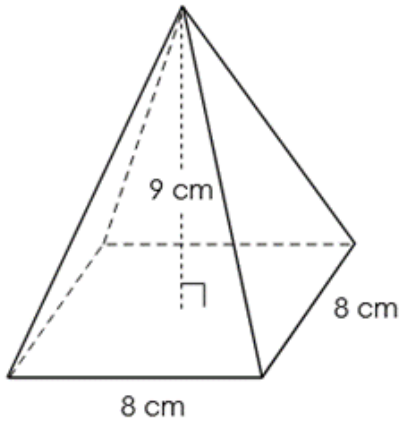

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct volume of the pyramid.

To calculate the volume of a pyramid, use the formula $V = \frac{1}{3}BH$, where B is the area of the base and H is the height of the pyramid. The base of the pyramid is a square. So, the area of the square is $B = 8 \cdot 8$ or 64 $cm^2$. Thus, $V = \frac{1}{3} \cdot 64 \cdot 9 = 192$ $cm^3$.

## Sample Response: 0 points

A square pyramid is shown, with dimensions in centimeters (cm).




What is the volume, in cubic centimeters, of the pyramid?

24 *cubic centimeters*

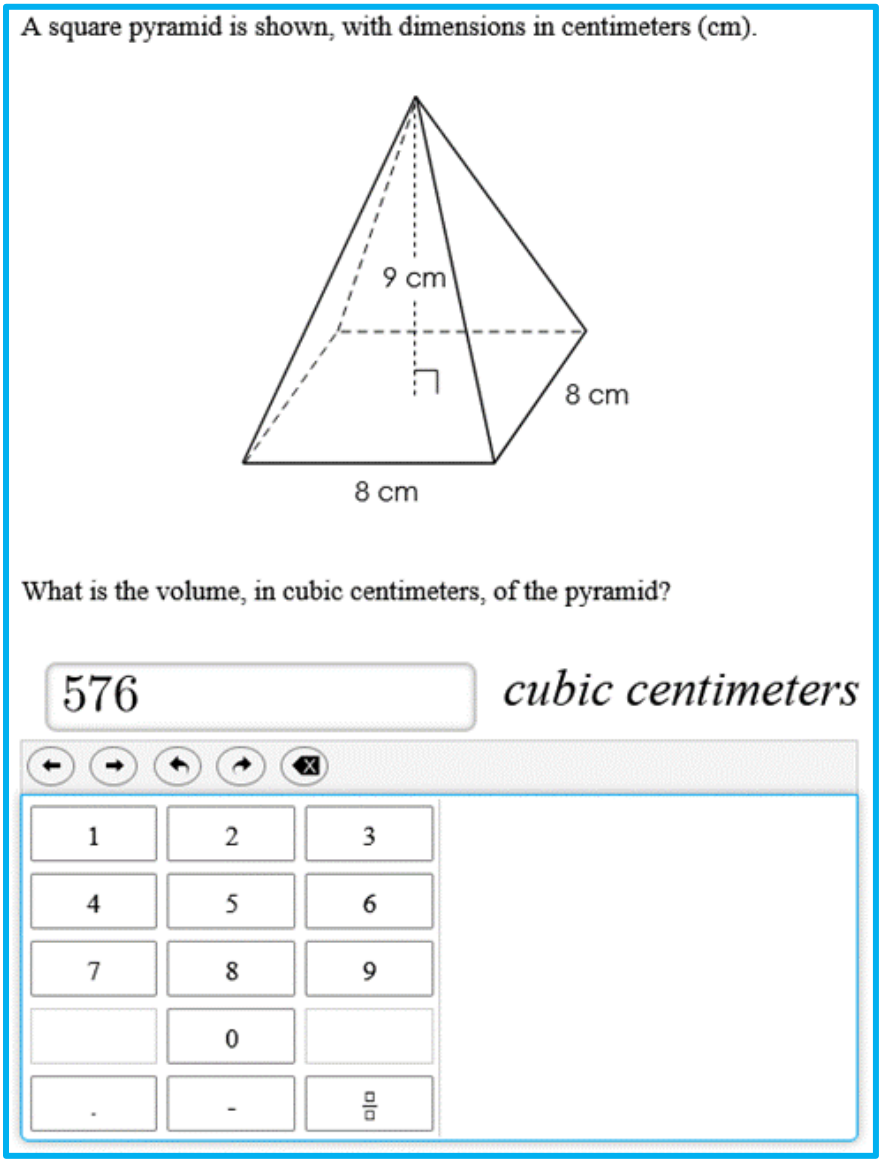

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect volume of a pyramid.

V = $\frac{1}{3}BH$, where B is the area of the base and H is the height of the pyramid. The student may have used the side length of the base of the pyramid, instead of the area of the base.

B = 8. Thus, V = $\frac{1}{3} \cdot 8 \cdot 9 = \frac{1}{3} \cdot 72 = 24$ cm$^3$.

## Sample Response: 0 points




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect volume of a pyramid.

The student may calculate the volume of the prism, using a formula V = $BH$ or V = $8 \cdot 8 \cdot 9$ instead of the volume of a pyramid using a formula V = $\frac{1}{3}BH$.

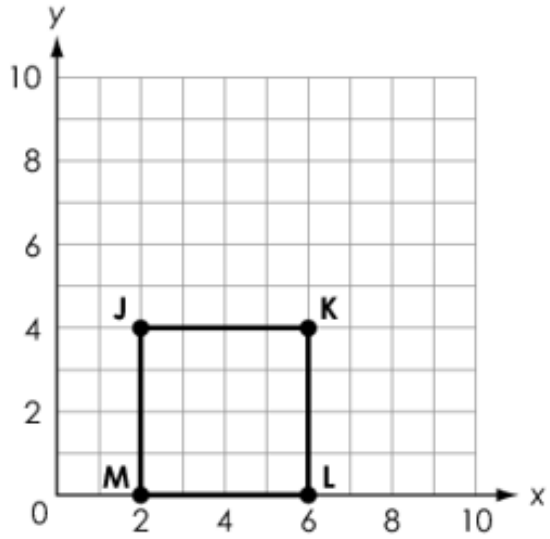
# Geometry
# Spring 2022 Item Release

# Question 22

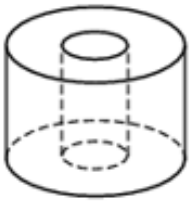
# Question and Scoring Guidelines

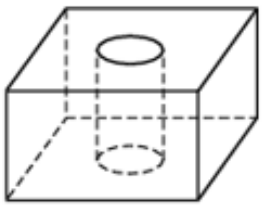
# Question 22

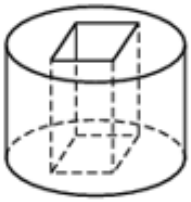
Square JKLM is shown.

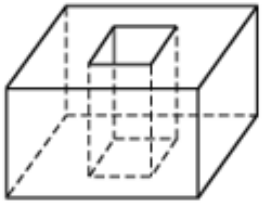



Which three-dimensional figure could result from rotating square JKLM 360° clockwise about the $y$-axis?

Ⓐ 

Ⓒ 

Ⓑ 

Ⓓ 

**Points Possible:** 1

**Content Cluster:** Visualize relationships between two-dimensional and three-dimensional objects.

**Content Standard:** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. *(G.GMD.4)*

**Depth of Knowledge:** Level 2
b. Interpret information from a simple graph
c. Use models to represent mathematical concepts
e. Compare and/or contrast figures or statements

# Scoring Guidelines

Rationale for Option A: **Key** – The student realizes the rotation of the two horizontal sides of the square about the *y*-axis generates two congruent horizontal circular bases with a hole with a radius of 2 units through the center of the solid. Since the vertical sides of the square are 2 and 6 units away from and parallel to the *y*-axis, the two vertical sides of the square generate two non-congruent curved surfaces 4 units away from each other.

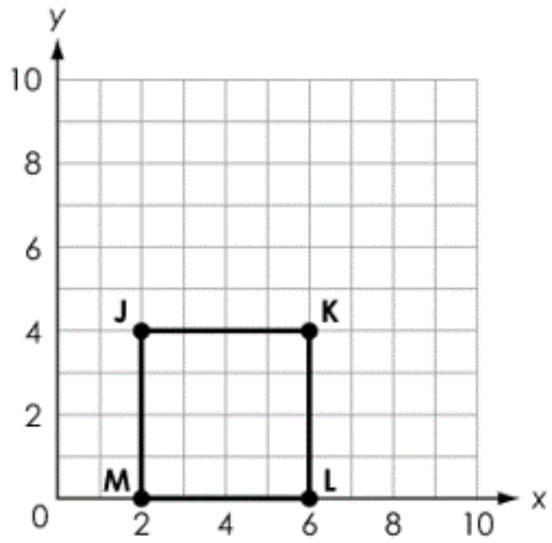
Rationale for Option B: This is incorrect. The student may incorrectly think that side JM creates a hole with a circular base, and thinks the hole is in the shape of a rectangular prism.

Rationale for Option C: This is incorrect. The student may realize that the rotation of the square about the *y*-axis creates a hole in the middle of the object, but incorrectly think that the three-dimensional hole may have corners like the square and be in the shape of a rectangular prism.

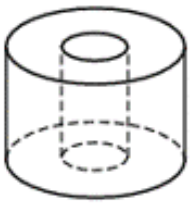
Rationale for Option D: This is incorrect. The student may not realize that a figure of rotation should have at least one curved surface.

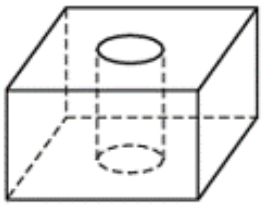
## Sample Response: 1 point

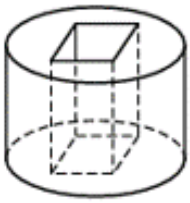
Square JKLM is shown.

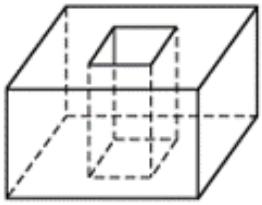



Which three-dimensional figure could result from rotating square JKLM 360° clockwise about the $y$-axis?

● 

Ⓒ 

Ⓑ 

Ⓓ 

# Geometry
# Spring 2022 Item Release

# Question 24

# Question and Scoring Guidelines

# Question 24

Westville has a land area of 84.4 square miles. In 2015, the population density was 22.5 people per square mile.

In 2016, the population was 2,000 people.

By how many people did the population of Westville increase between 2015 and 2016?


**Points Possible:** 1

**Content Cluster:** Apply geometric concepts in modeling situations.

**Content Standard:** Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.★ *(G.MG.2)*

**Depth of Knowledge:** Level 2
I. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

- 101

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

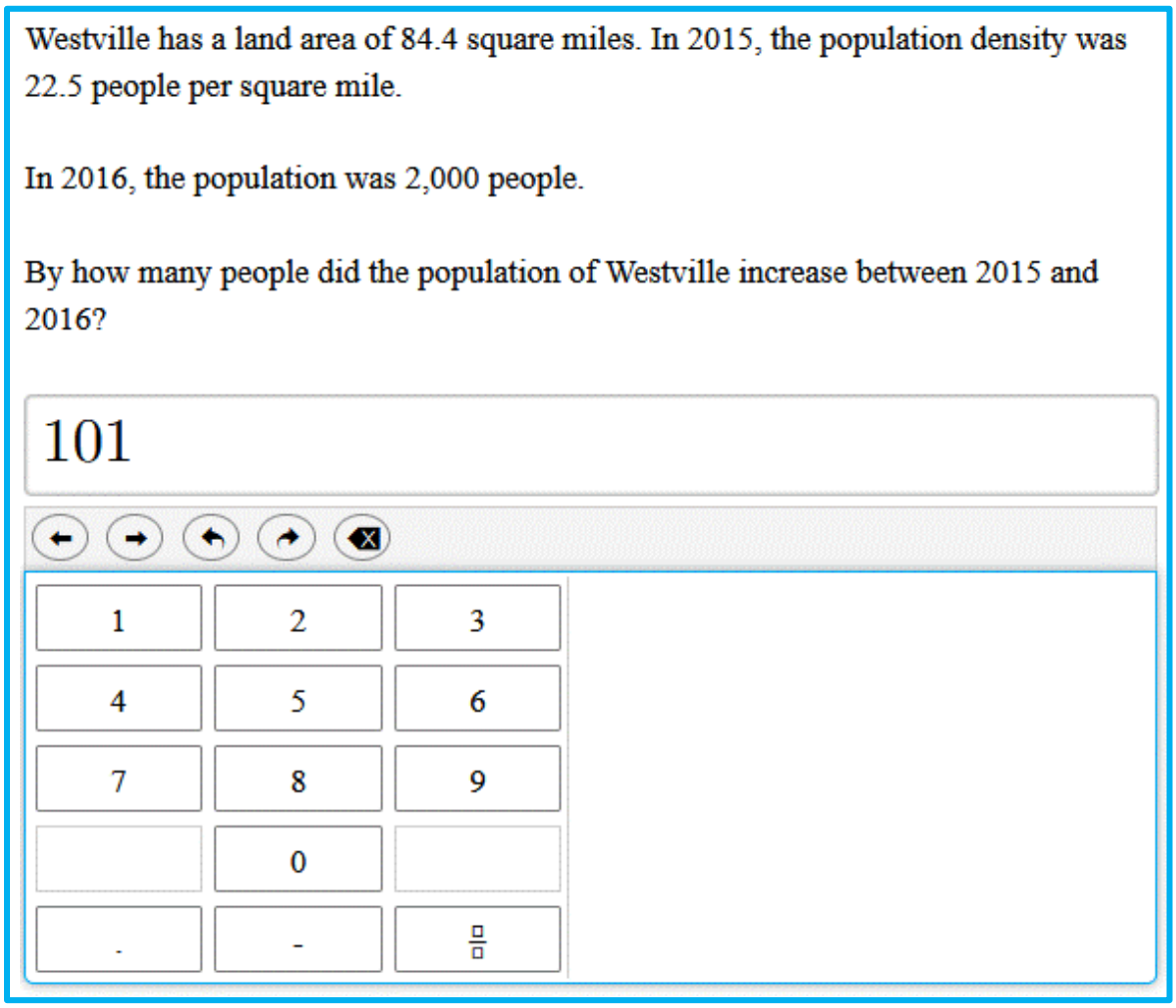
- the correct number of people (1 point).

# Geometry
# Spring 2022 Item Release

# Question 24

# Sample Responses

## Sample Response: 1 point



**Notes on Scoring**

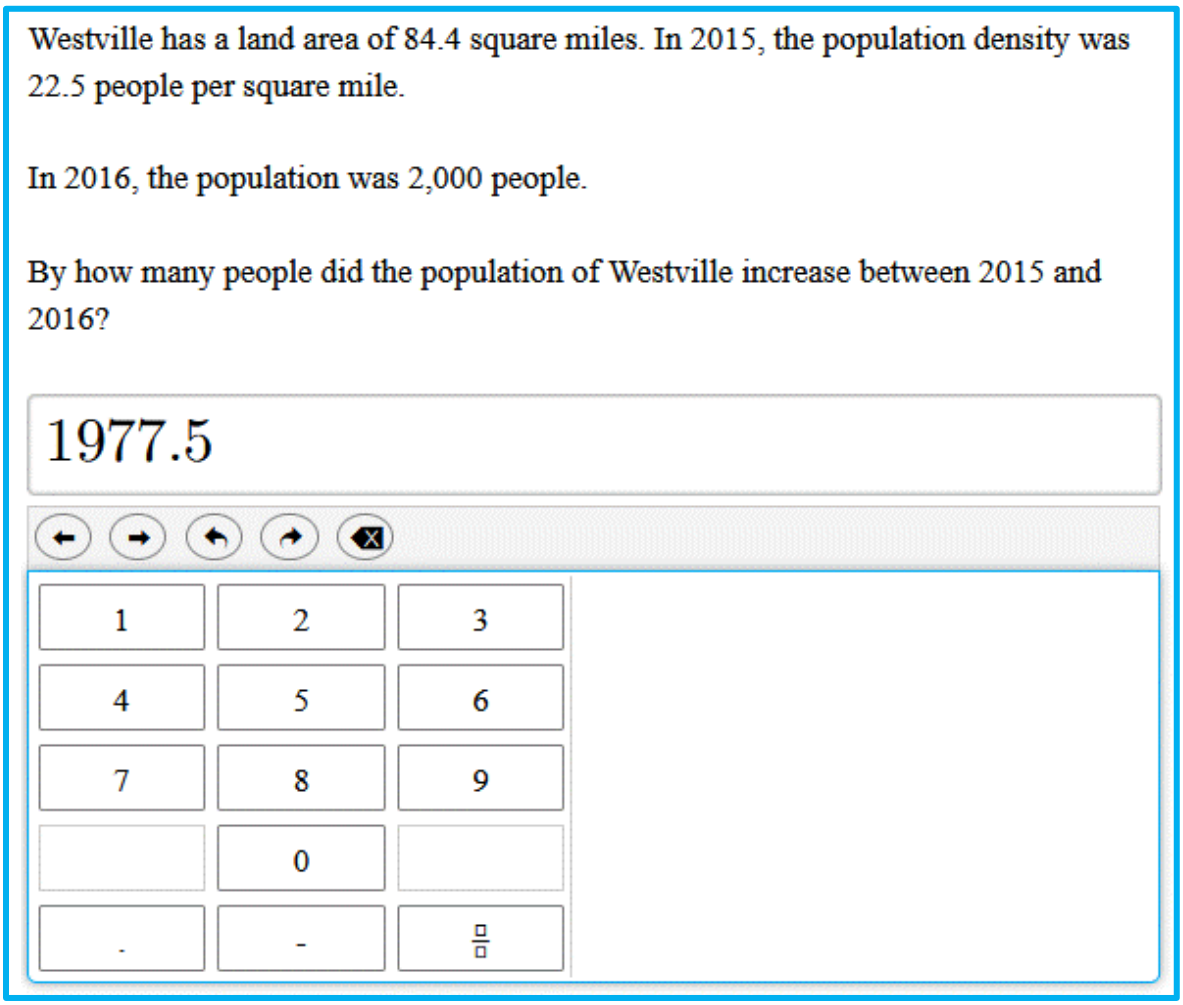
This response earns full credit (1 point) because it shows a correct increase in population.

To find by how many people the population of Westerville increased between 2015 and 2016, find the population in 2015 using the formula for the density of the population D = $\frac{Population}{Area}$.

In this situation, the density of the population is 22.5 people per square mile, the population in 2015 is x people and the land area is 84.4 square miles. Substitute these values in the formula to solve the equation for x as $22.5 = \frac{x}{84.4}$ or $x = 22.5 \cdot 84.4 = 1899$. So, the population of Westerville in 2015 was 1899 people.

If the population of Westerville in 2016 was 2000 people, the increase between 2015 and 2016 was 2000 – 1899 = 101 people.

## Sample Response: 0 points

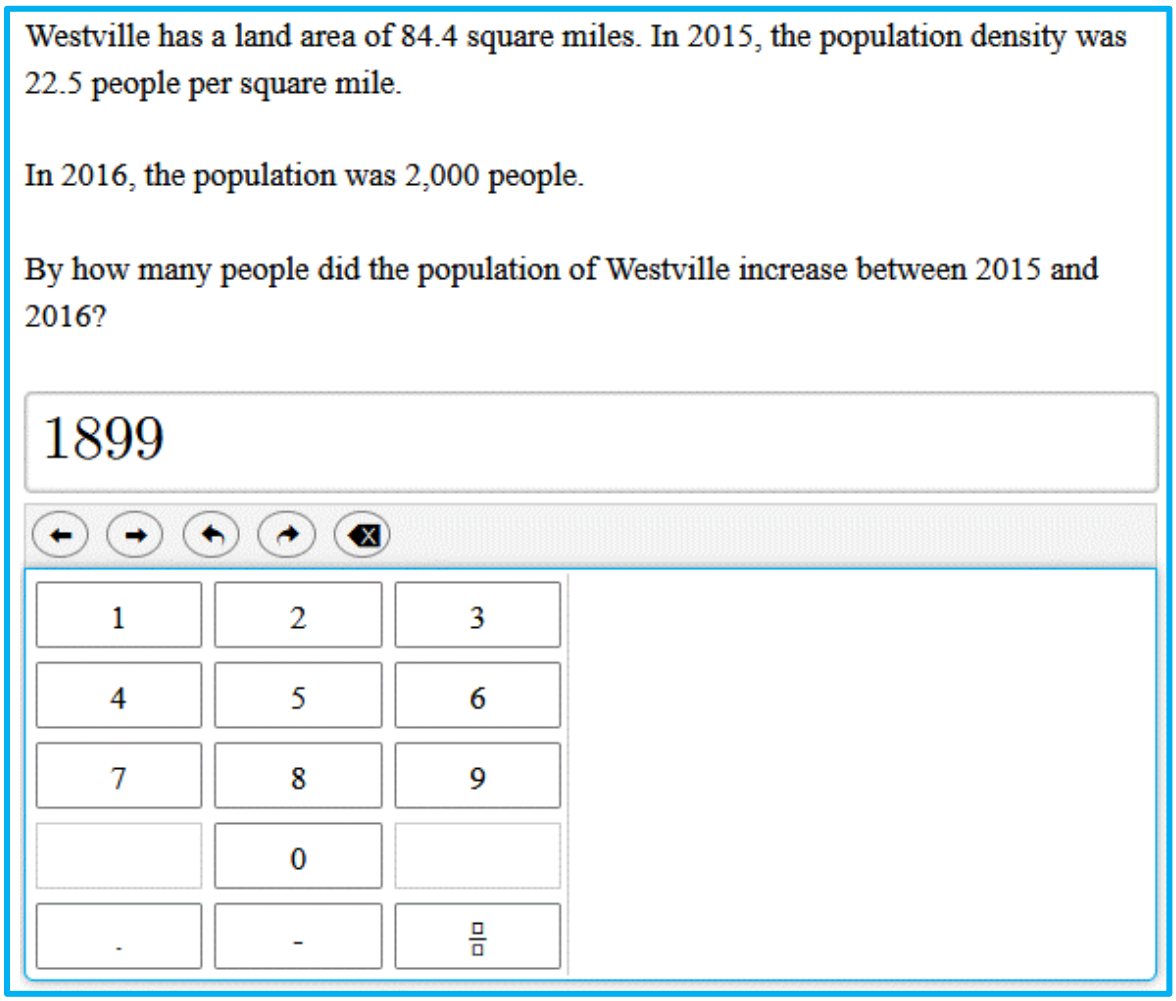



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect increase in population.

The student may fail to find the population of Westerville in 2015. Instead, the student may subtract 22.5 people per square mile from the population in Westerville in 2016, or 2,000 – 22.5, to get 1977.5.

## Sample Response: 0 points




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect increase in population.

Since the density of the population is 22.5 people per square mile, and the land area is 84.4 square miles, the population of Westerville in 2015 is 22.5 · 84.4 = 1899 people. The student may use this number for the answer without subtracting it from 2,000 to determine how much the population increased from 2015 to 2016.

# Geometry
# Spring 2022 Item Release

# Question 26

# Question and Scoring Guidelines

# Question 26

A company surveys people in a city to find the type of cell phone plans they have and which provider they use. The results are shown in the table.

| | Unlimited Usage | Limited Usage | Total |
|---|---|---|---|
| **Provider A** | 19 | 4 | 23 |
| **Provider B** | 14 | 6 | 20 |
| **Provider C** | 13 | 11 | 24 |
| **Other** | 7 | 1 | 8 |
| **Total** | 53 | 22 | 75 |

What is the probability that a randomly selected person who has a plan with Provider B has unlimited usage?

**Points Possible:** 1

**Content Cluster:** Understand independence and conditional probability, and use them to interpret data.

**Content Standard:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.★ (S.CP.4)*

**Depth of Knowledge:** Level 2
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

- 0.7

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

- the correct probability (1 point).

# Geometry
# Spring 2022 Item Release

# Question 26

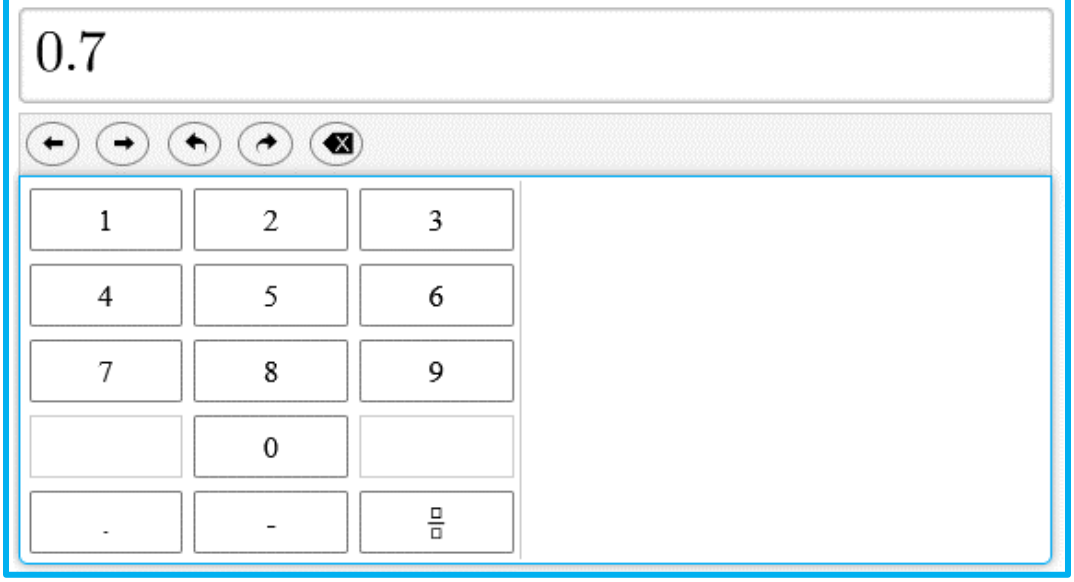
# Sample Responses

## Sample Response: 1 point

A company surveys people in a city to find the type of cell phone plans they have and which provider they use. The results are shown in the table.

| | Unlimited Usage | Limited Usage | Total |
|---|---|---|---|
| **Provider A** | 19 | 4 | 23 |
| **Provider B** | 14 | 6 | 20 |
| **Provider C** | 13 | 11 | 24 |
| **Other** | 7 | 1 | 8 |
| **Total** | 53 | 22 | 75 |

What is the probability that a randomly selected person who has a plan with Provider B has unlimited usage?



**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct probability.

The situation requires the ability to interpret information summarized in the table. The number of people that have Provider B is 20 and the number of people who have unlimited usage with Provider B is 14. Therefore, the probability that a randomly selected person has a plan with unlimited usage if they have Provider B is $\frac{14}{20}$ or 0.7.

## Sample Response: 1 point

A company surveys people in a city to find the type of cell phone plans they have and which provider they use. The results are shown in the table.

| | Unlimited Usage | Limited Usage | Total |
|---|---|---|---|
| **Provider A** | 19 | 4 | 23 |
| **Provider B** | 14 | 6 | 20 |
| **Provider C** | 13 | 11 | 24 |
| **Other** | 7 | 1 | 8 |
| **Total** | 53 | 22 | 75 |

What is the probability that a randomly selected person who has a plan with Provider B has unlimited usage?

$\frac{7}{10}$

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct probability in equivalent form.

The situation requires the ability to interpret information summarized in the table. The number of people that have Provider B is 20 and the number of people who have unlimited usage with Provider B is 14. Therefore, the probability that a randomly selected person has a plan with unlimited usage if they have Provider B is $\frac{14}{20}$ $or$ $\frac{7}{10}$.

## Sample Response: 0 points

A company surveys people in a city to find the type of cell phone plans they have and which provider they use. The results are shown in the table.

| | Unlimited Usage | Limited Usage | Total |
|---|---|---|---|
| **Provider A** | 19 | 4 | 23 |
| **Provider B** | 14 | 6 | 20 |
| **Provider C** | 13 | 11 | 24 |
| **Other** | 7 | 1 | 8 |
| **Total** | 53 | 22 | 75 |

What is the probability that a randomly selected person who has a plan with Provider B has unlimited usage?

$\frac{14}{75}$

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect probability.

The student recognizes that 14 people have Provider B and unlimited usage but may then use the total number of people in the survey, or 75, to get a probability of $\frac{14}{75}$ instead of the total number of people with Provider B, or 20, to get a probability of $\frac{14}{20}$.

## Sample Response: 0 points

A company surveys people in a city to find the type of cell phone plans they have and which provider they use. The results are shown in the table.

| | Unlimited Usage | Limited Usage | Total |
|---|---|---|---|
| **Provider A** | 19 | 4 | 23 |
| **Provider B** | 14 | 6 | 20 |
| **Provider C** | 13 | 11 | 24 |
| **Other** | 7 | 1 | 8 |
| **Total** | 53 | 22 | 75 |

What is the probability that a randomly selected person who has a plan with Provider B has unlimited usage?

$\frac{14}{53}$

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect probability.

The student recognizes that 14 people have Provider B and unlimited usage but may then use the total number of people who have unlimited usage, or 53, to get a probability of $\frac{14}{53}$ instead of the total number of people with Provider B, or 20, to get a probability of $\frac{14}{20}$.

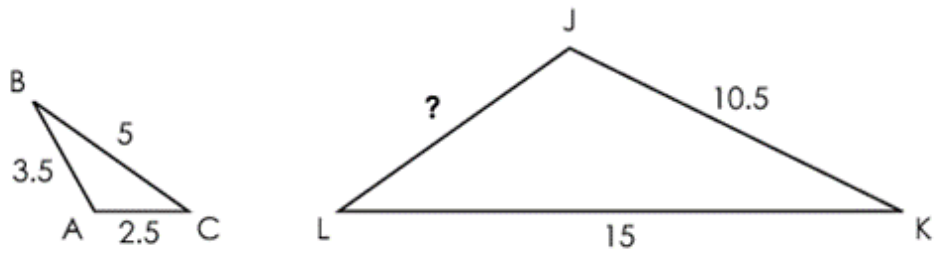
# Geometry
# Spring 2022 Item Release

# Question 27

# Question and Scoring Guidelines

# Question 27

Two similar triangles, Δ ABC and Δ JKL, are shown.




What is the length of side JL?

**Points Possible:** 1

**Content Cluster:** Prove and apply theorems both formally and informally involving similarity using a variety of methods.

**Content Standard:** Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles. *(G.SRT.5)*

**Depth of Knowledge:** Level 2
c. Use models to represent mathematical concepts
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

- 7.5

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

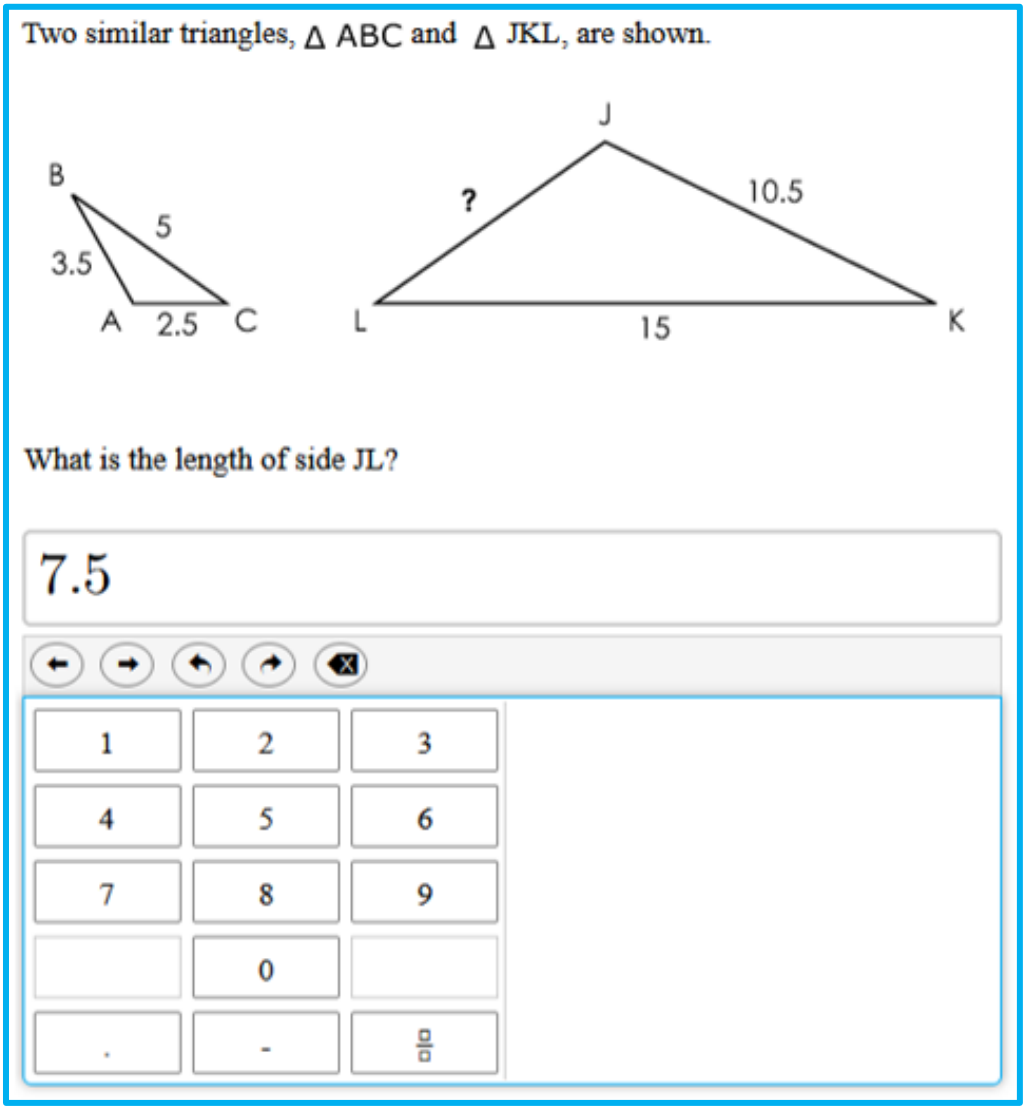
- a correct value (1 point).

# Geometry
# Spring 2022 Item Release

# Question 27

# Sample Responses

## Sample Response: 1 point

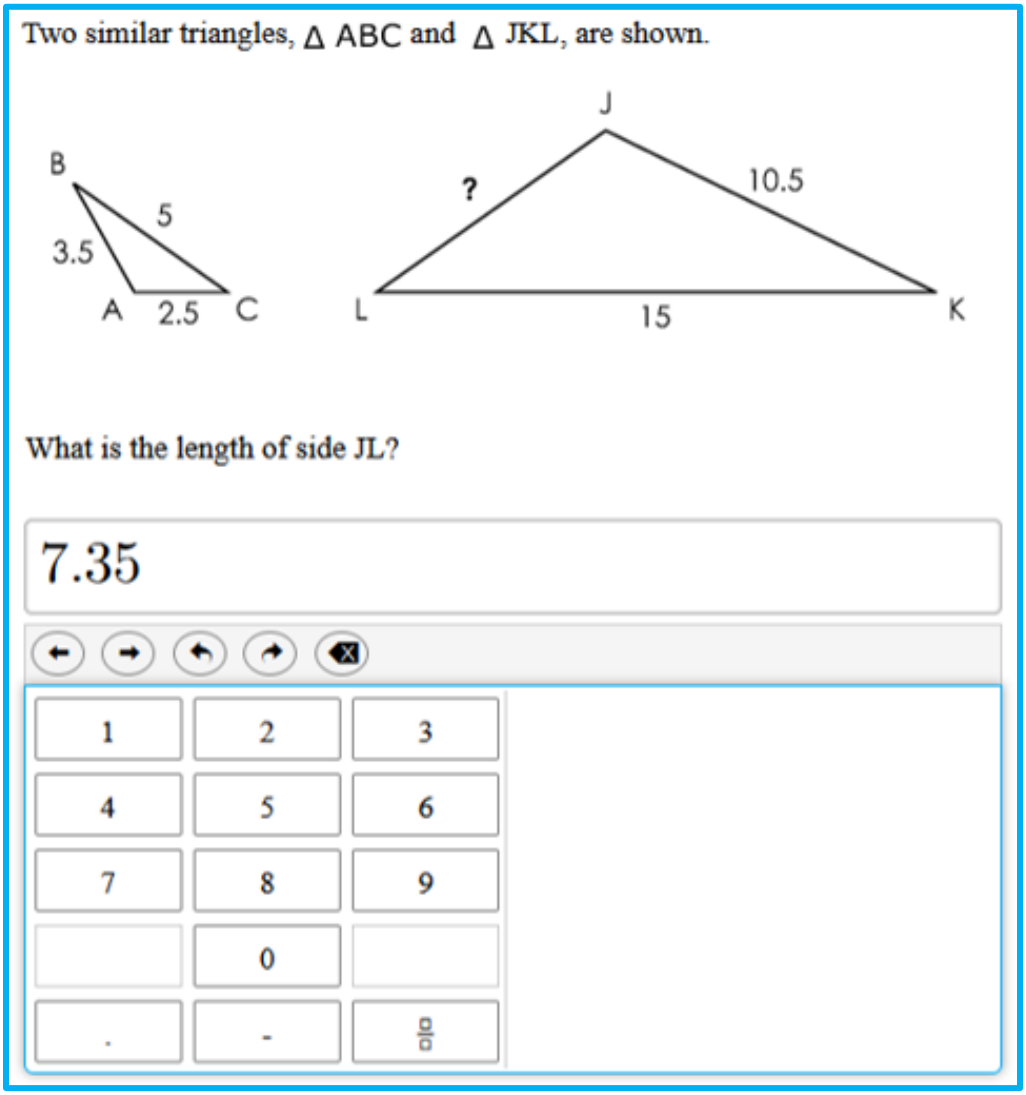



**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct side length of the triangle.

Since triangles ABC and JKL are similar, their corresponding sides are proportional, or $\frac{KL}{BC} = \frac{JK}{AB} = \frac{JL}{AC}$. After substituting the side lengths, or $\frac{15}{5} = \frac{10.5}{3.5} = \frac{JL}{2.5}$, use the ratio $\frac{JL}{2.5}$ and any of the other two ratios to solve a proportion for the unknown JL. For example, $\frac{15}{5} = \frac{JL}{2.5}$. Multiply both sides by 2.5 to get $\frac{15}{5} \cdot 2.5 = JL$ or $JL = 7.5$.

## Sample Response: 0 points

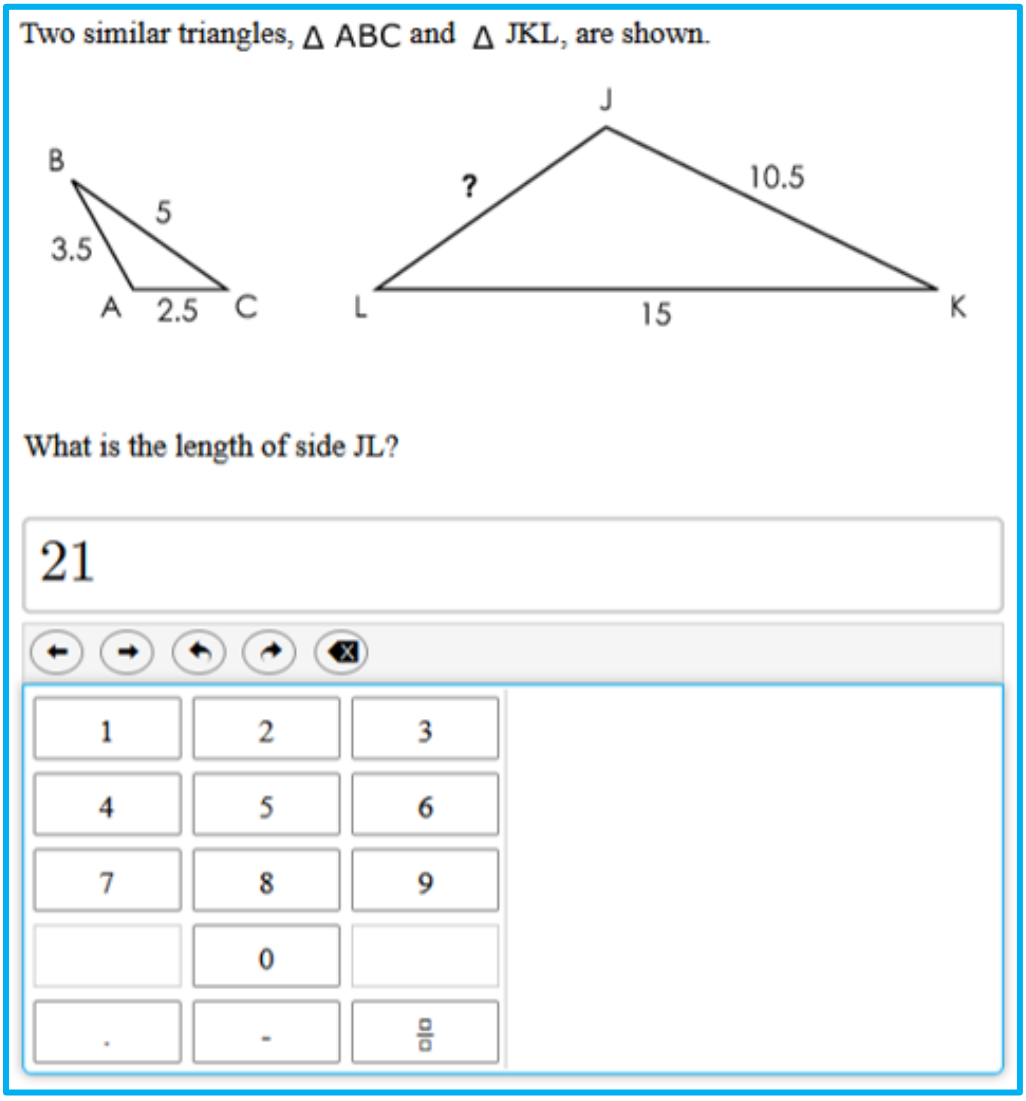



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length of side JL.

The student may fail to find correct corresponding sides of the triangles, and thus creates an incorrect proportion $\frac{3.5}{JL} = \frac{5}{10.5}$ instead of $\frac{2.5}{JL} = \frac{5}{15.}$. Next, the student may solve this incorrect proportion as $JL = \frac{3.5 \cdot 10.5}{5} = 7.35$.

## Sample Response: 0 points




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length of side JL.

The student may fail to find correct corresponding sides of the triangles, and thus creates an incorrect proportion $\frac{3.5}{JL} = \frac{2.5}{15}$ instead of $\frac{2.5}{JL} = \frac{5}{15.}$. Next, the student may solve this incorrect proportion as $JL = \frac{3.5 \cdot 15}{2.5} = 21$.

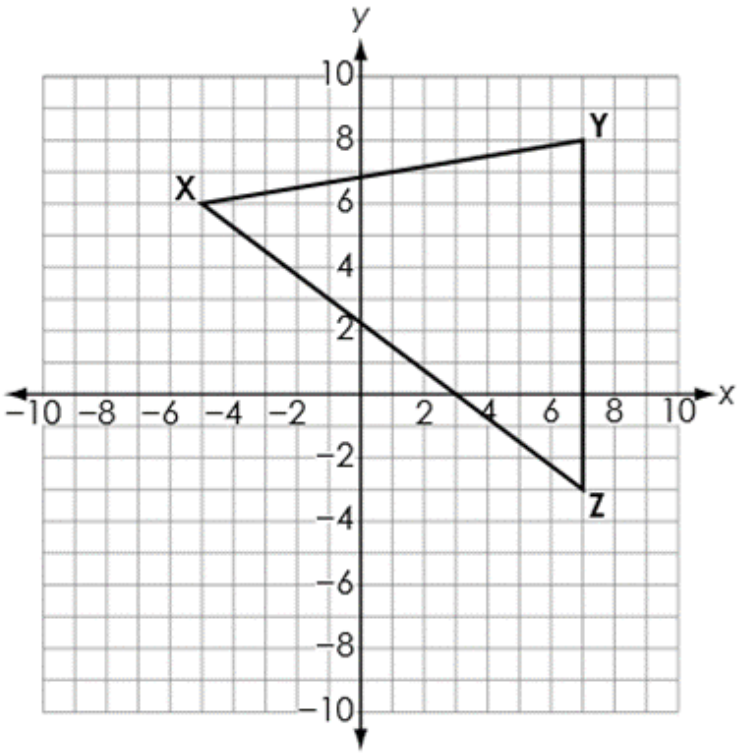
# Geometry
# Spring 2022 Item Release

# Question 28

# Question and Scoring Guidelines

## Question 28

Triangle XYZ is shown.




Which transformation on $\Delta XYZ$ results in a triangle that is similar, but not congruent, to $\Delta XYZ$?

(A) a translation 3 units to the right

(B) a reflection across the line $y = -2x + 3$

(C) a rotation 90° clockwise around the origin

(D) a dilation centered at the origin with a scale factor of 2

**Points Possible:** 1

**Content Cluster:** Understand similarity in terms of similarity transformations.

**Content Standard:** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. *(G.SRT.2)*

**Depth of Knowledge:** Level 1
a. Recall, observe, or recognize a fact, definition, term, or property
n. Represent math relationships in words, pictures, or symbols

# Scoring Guidelines

Rationale for Option A: This is incorrect. The student may assume that because of the translation the coordinates of the vertices X, Y and Z would change, so that the translated image of ΔXYZ is not congruent to ΔXYZ and the triangles are similar, but not congruent.

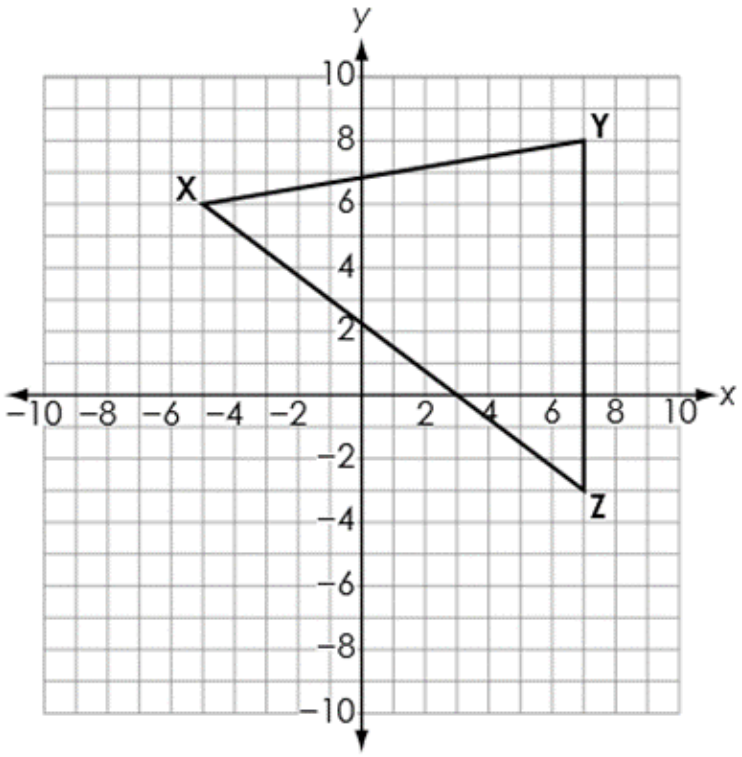
Rationale for Option B: This is incorrect. The student may assume that because of the reflection across the line $y = -2x + 3$ the coordinates of the vertices X, Y and Z would change, so that the reflected image of ΔXYZ and ΔXYZ are similar, but not congruent.

Rationale for Option C: This is incorrect. The student may assume that because of the 90° rotation the coordinates of the vertices X, Y and Z would change, so that the rotated image of ΔXYZ and ΔXYZ are similar, but not congruent.

Rationale for Option D: **Key** – The student correctly identifies that a dilation with a scale factor greater than one creates a triangle that is larger than triangle XYZ with proportional sides and angles congruent to the corresponding angles of the triangle XYZ. Therefore, the result of the transformation of triangle XYZ is similar but not congruent to triangle XYZ.

## Sample Response: 1 point

Triangle XYZ is shown.




Which transformation on $\Delta XYZ$ results in a triangle that is similar, but not congruent, to $\Delta XYZ$?

Ⓐ a translation 3 units to the right

Ⓑ a reflection across the line $y = -2x + 3$

Ⓒ a rotation 90° clockwise around the origin

● a dilation centered at the origin with a scale factor of 2

# Geometry
# Spring 2022 Item Release

# Question 30

# Question and Scoring Guidelines

# Question 30

The volume of a cylinder is 90 cubic centimeters.

Which step can be performed to find the volume of a cone with the same radius and height as the cylinder?

(A) divide the volume of the cylinder by $\pi$

(B) divide the volume of the cylinder by 3

(C) divide the volume of the cylinder by the height

(D) divide the volume of the cylinder by the radius

**Points Possible:** 1

**Content Cluster:** Explain volume formulas, and use them to solve problems.

**Content Standard:** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ *(G.GMD.3)*

**Depth of Knowledge:** Level 2
e. Compare and/or contrast figures or statements
j. Translate between tables, graphs, words and symbolic notation
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Rationale for Option A: This is incorrect. The student may correctly realize that the volume of a cylinder can be found as $\pi r^2h$ but incorrectly think that the volume of a cone is $V = r^2h$ and note that dividing the volume of the cylinder, $\pi r^2h$, by $\pi$ would lead to $V = r^2h$.

Rationale for Option B: **Key** – The student correctly realizes that the volume of a cylinder with radius $r$ and height $h$ can be calculated as $V = \pi r^2h$ and the volume of a cone with the same radius $r$ and height $h$ can be calculated as $V = \frac{1}{3}\pi r^2h$, dividing the volume of a cylinder by 3 will result in the volume of a cone.

Rationale for Option C: This is incorrect. The student may correctly realize that the volume of a cylinder can be calculated as $V = \pi r^2h$ but may incorrectly think that the volume of a cone is $V = \pi r^2$ to conclude that dividing the volume of the cylinder, $\pi r^2h$, by $h$ would result in $\pi r^2$.

Rationale for Option D: This is incorrect. The student may correctly realize that the volume of a cylinder can be calculated as $V = \pi r^2h$ but may incorrectly think that the volume of a cone is $V = \pi r^2$ to conclude that dividing the volume of the cylinder, $\pi r^2h$, by $r$ would result in $\pi r^2$.

## Sample Response: 1 point

The volume of a cylinder is 90 cubic centimeters.

Which step can be performed to find the volume of a cone with the same radius and height as the cylinder?

- (A) divide the volume of the cylinder by π
- ● divide the volume of the cylinder by 3
- (C) divide the volume of the cylinder by the height
- (D) divide the volume of the cylinder by the radius

# Geometry
# Spring 2022 Item Release

# Question 31

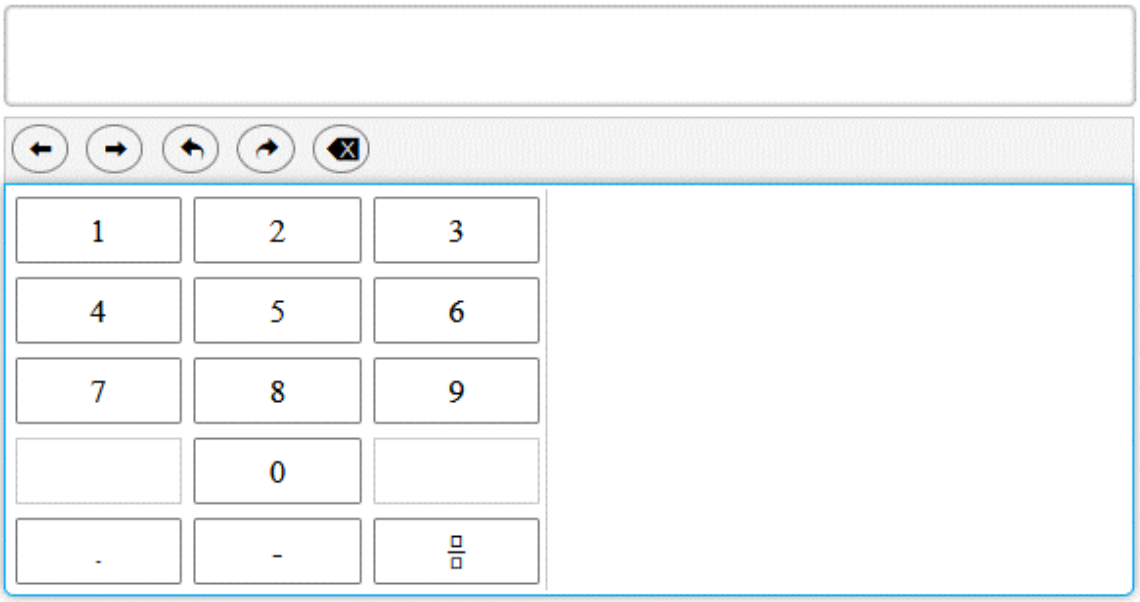
# Question and Scoring Guidelines

# Question 31

Hernan randomly selects a book from his library. He calculates the probabilities given.

- $P(\text{hardcover or fiction}) = 0.8$
- $P(\text{hardcover and fiction}) = 0.2$
- $P(\text{hardcover}) = 0.6$

What is the probability that Hernan selects a fiction book?



**Points Possible:** 1

**Content Cluster:** Use the rules of probability to compute probabilities of compound events in a uniform probability model.

**Content Standard:** Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★ *(S.CP.7)*

**Depth of Knowledge:** Level 2
k. Make direct translations between problem situations and symbolic notation
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

- 0.4

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

- a correct probability (1 point).

# Geometry
# Spring 2022 Item Release

# Question 31

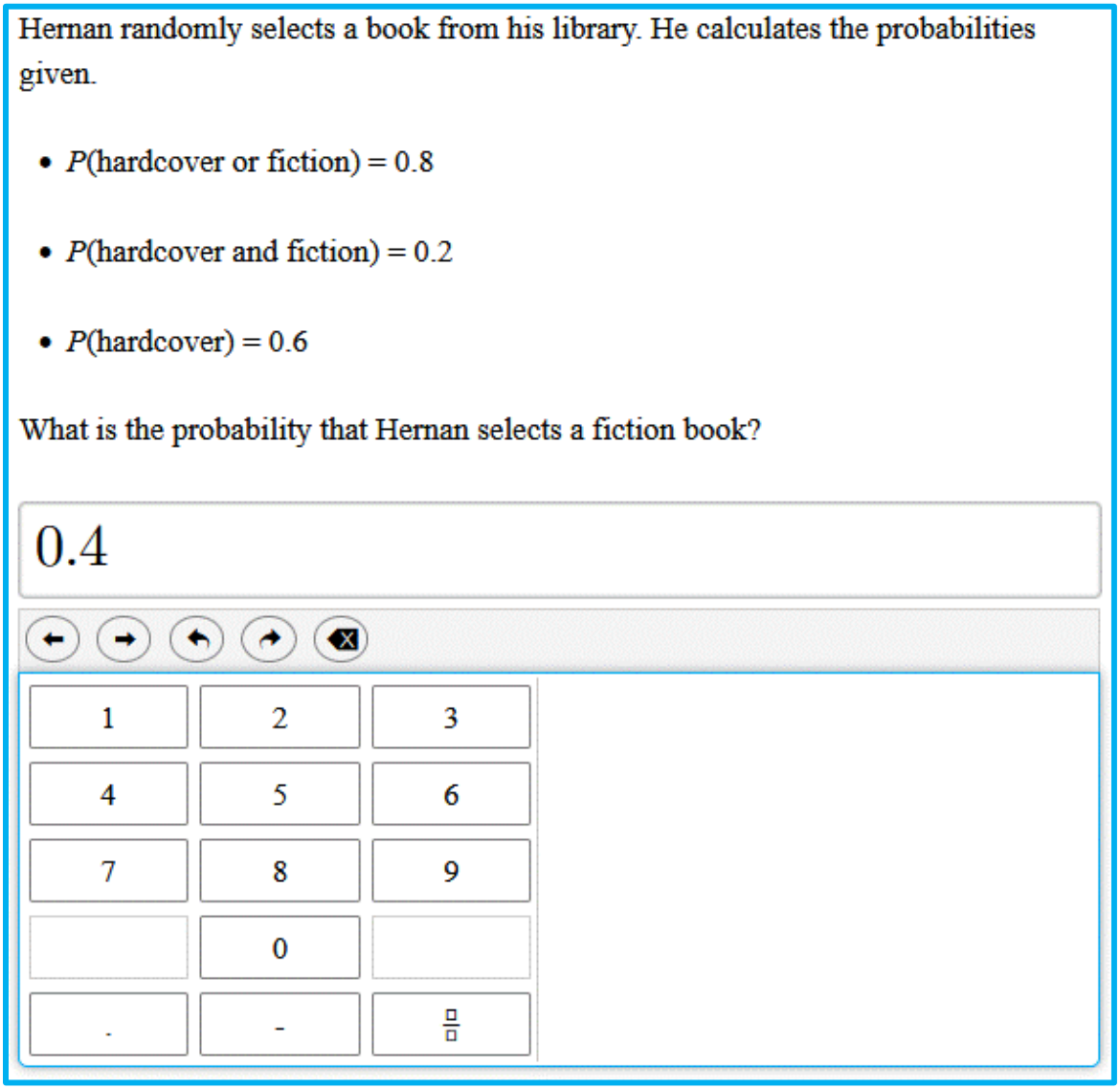
# Sample Responses

## Sample Response: 1 point

Hernan randomly selects a book from his library. He calculates the probabilities given.

- $P(\text{hardcover or fiction}) = 0.8$
- $P(\text{hardcover and fiction}) = 0.2$
- $P(\text{hardcover}) = 0.6$

What is the probability that Hernan selects a fiction book?



**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct probability of the event.

The probability of a randomly selected hardcover or fiction book from the library can be calculated using the Addition Rule for the probability of compound events, or
$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$.

The probability of a randomly selected hard cover or fiction book is $P$(hardcover ***or*** fiction) = 0.8. The probability of a randomly selected hard cover and fiction is $P$(hardcover **and** fiction) = 0.2. The probability of a randomly selected hardcover book is $P$(hardcover) = 0.6.

By substituting these values into the Addition Rule formula, $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$, find the unknown probability of a randomly selected fiction book, P(B), as

$0.8 = 0.6 + P(B) - 0.2$

$0.8 = 0.4 + P(B)$

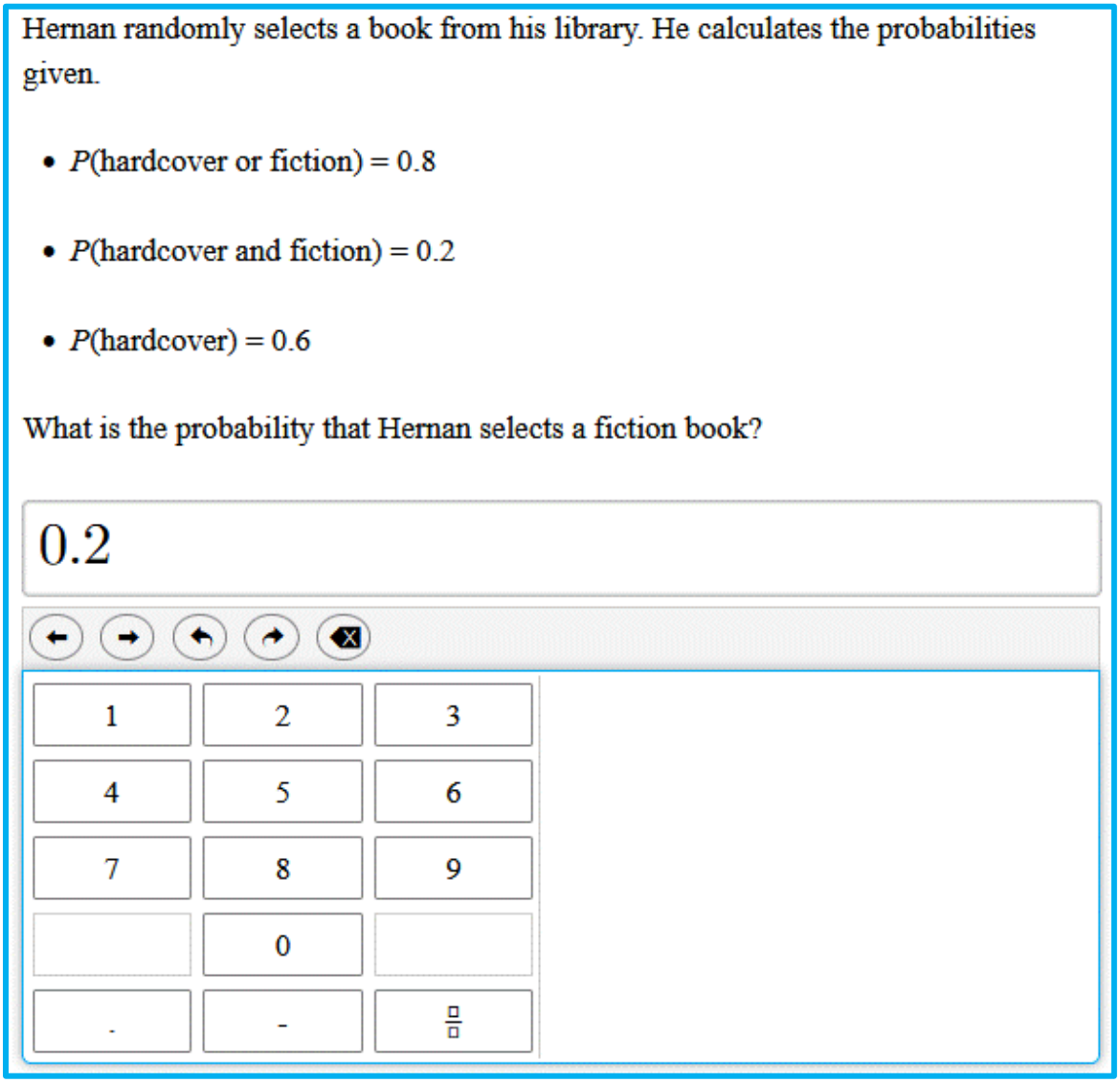
$0.4 = P(B)$

## Sample Response: 0 points

Hernan randomly selects a book from his library. He calculates the probabilities given.

- $P(\text{hardcover or fiction}) = 0.8$
- $P(\text{hardcover and fiction}) = 0.2$
- $P(\text{hardcover}) = 0.6$

What is the probability that Hernan selects a fiction book?




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect probability of selecting a fiction book.

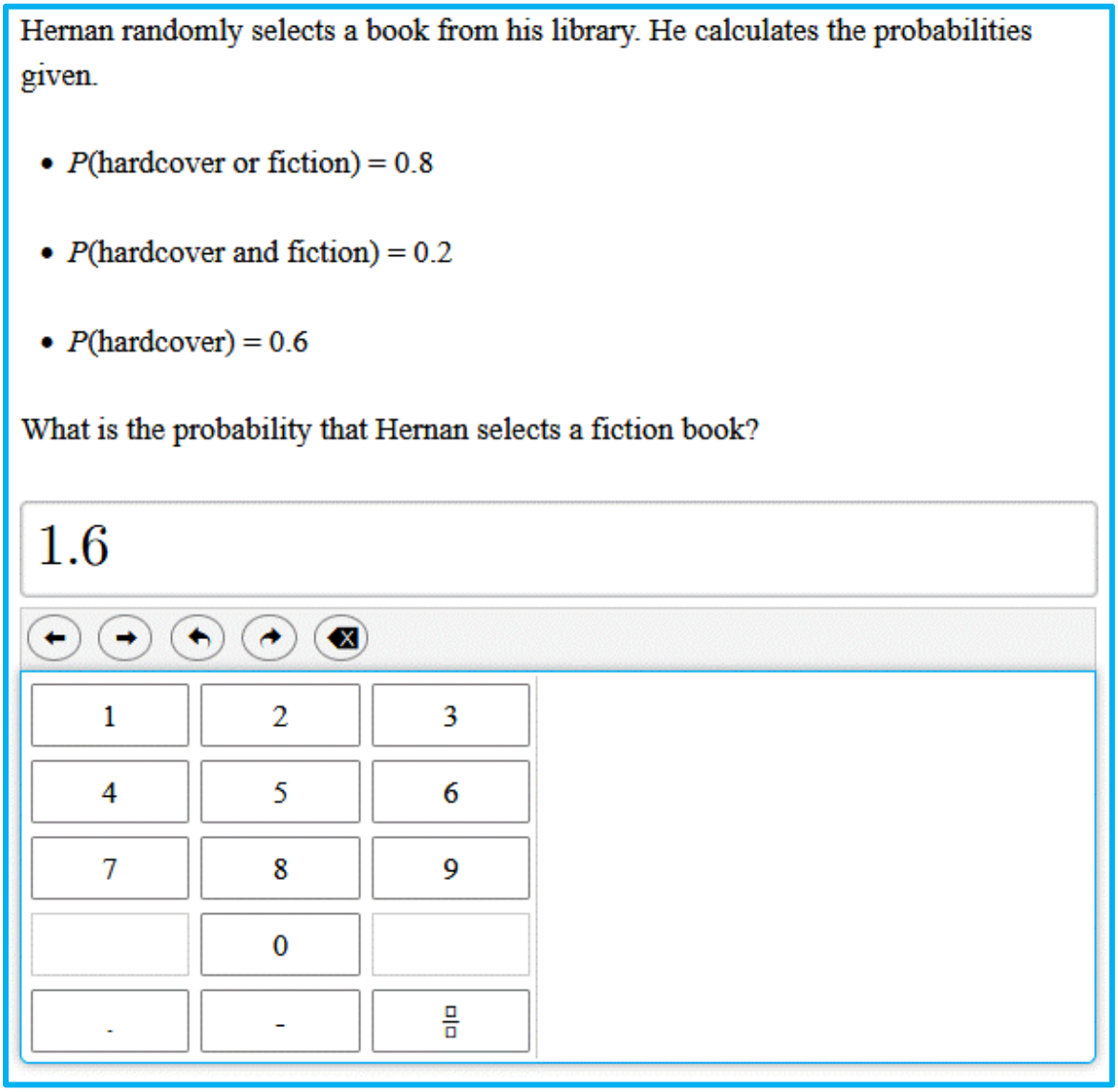
The student may subtract the probability of selecting a hardcover book or a fiction book, or 0.8, and the probability of selecting just a hardcover book, or 0.6, to get 0.2 instead of using the Addition Rule to find the probability that Hernan selects a fiction book.

## Sample Response: 0 points

Hernan randomly selects a book from his library. He calculates the probabilities given.

- $P$(hardcover or fiction) = 0.8
- $P$(hardcover and fiction) = 0.2
- $P$(hardcover) = 0.6

What is the probability that Hernan selects a fiction book?




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect probability of selecting a fiction book.

The student may add all three probabilities, $0.8 + 0.6 + 0.2 = 1.6$, instead of using the Addition Rule to find the probability that Hernan selects a fiction book.

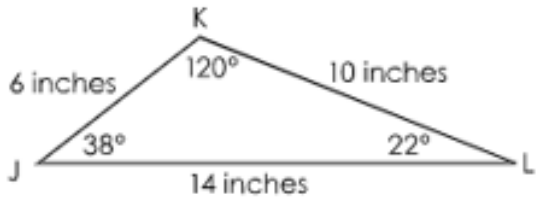
# Geometry
# Spring 2022 Item Release

# Question 32

# Question and Scoring Guidelines

## Question 32

Megan dilates $\triangle JKL$ about point J by a scale factor of $\frac{3}{5}$ to create $\triangle JNO$.




What is the length, in inches, of $\overline{NO}$ ?

$NO =$ _____ *inches*

1 2 3
4 5 6
7 8 9
0
. -

What is the measure, in degrees, of $\angle O$ ?

_____ *degrees*

1 2 3
4 5 6
7 8 9
0
. -

**Points Possible:** 2

**Content Cluster:** Understand similarity in terms of similarity transformations.

**Content Standard:** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. *(G.SRT.2)*

**Depth of Knowledge:** Level 2
b. Interpret information from a simple graph
k. Make direct translations between problem situations and symbolic notation
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

- *NO* = 6 inches
- 22 degrees

Other Correct Responses

- any equivalent values

For this item, a full-credit response includes:

- a correct length (1 point)

  and

- a correct angle measure (1 point).

For this item, a partial-credit response includes:

- a correct length (1 point)

  OR

- a correct angle measure (1 point).

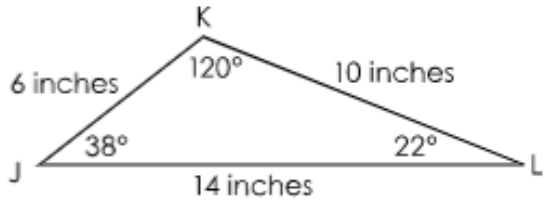
# Geometry
# Spring 2022 Item Release

# Question 32

# Sample Responses

**Sample Response: 2 points**

Megan dilates $\Delta JKL$ about point J by a scale factor of $\frac{3}{5}$ to create $\Delta JNO$.




What is the length, in inches, of $\overline{NO}$ ?

$NO =$ 6 *inches*


What is the measure, in degrees, of $\angle O$ ?

22 *degrees*

**Notes on Scoring**

This response earns full credit (2 points) because it shows a correctly determined side length and the correct angle measure of the dilated triangle.

The triangles JKL and JNO are similar because Δ JNO is a result of dilation of Δ JKL by the scale factor $\frac{3}{5}$ making lengths of corresponding sides proportional with the constant scale factor $\frac{3}{5}$, or $\frac{\mathbf{JN}}{\mathbf{JK}} = \frac{\mathbf{NO}}{\mathbf{KL}} = \frac{\mathbf{JO}}{\mathbf{JL}} = \frac{\mathbf{3}}{\mathbf{5}}$.

Use $\frac{\mathbf{NO}}{\mathbf{KL}} = \frac{\mathbf{3}}{\mathbf{5}}$ and KL = 10 to find the unknown side length NO by solving a proportion as

$\frac{\boldsymbol{NO}}{\mathbf{10}} = \frac{\mathbf{3}}{\mathbf{5}}$ Multiply both sides of the equation by 50

$3 \cdot 10 = NO \cdot 5$

$30 = NO \cdot 5$ Divide both sides of the equation by 5

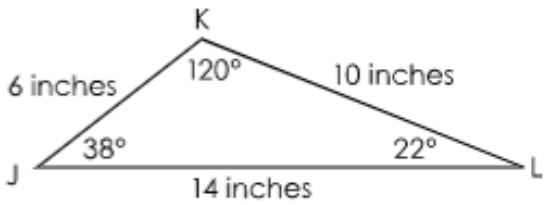
$6 = NO$

NO = 6 inches is a correct answer for the first question.

The correct answer to the second question is 22 degrees. Since the original and the scaled (dilated) triangle are similar, the corresponding angles of the triangles are congruent, and their measures are equal. So, the angles L and O are corresponding and therefore, $\angle O \cong \angle L$ and $m\angle O = m\angle L$. So, the $m\angle O = 22°$.

**Sample Response: 2 points**

Megan dilates $\Delta JKL$ about point J by a scale factor of $\frac{3}{5}$ to create $\Delta JNO$.




What is the length, in inches, of $\overline{NO}$ ?

$NO =$ $\frac{30}{5}$ *inches*

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | |

What is the measure, in degrees, of $\angle O$ ?

22.0 *degrees*

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | |

**Notes on Scoring**

This response earns full credit (2 points) because it shows a correctly determined side length in equivalent form and the correct equivalent form of the angle measure of the dilated triangle.

The triangles JKL and JNO are similar because Δ JNO is a result of dilation of Δ JKL by the scale factor $\frac{3}{5}$ making lengths of corresponding sides proportional with the constant scale factor $\frac{3}{5}$, or $\frac{\mathbf{JN}}{\mathbf{JK}} = \frac{\mathbf{NO}}{\mathbf{KL}} = \frac{\mathbf{JO}}{\mathbf{JL}} = \frac{\mathbf{3}}{\mathbf{5}}$.

Use $\frac{\mathbf{NO}}{\mathbf{KL}} = \frac{\mathbf{3}}{\mathbf{5}}$ and KL = 10 in to find the unknown side length NO by solving a proportion as

$\frac{\mathbf{NO}}{\mathbf{10}} = \frac{\mathbf{3}}{\mathbf{5}}$ Multiply both sides of the equation by 50

$3 \cdot 10 = NO \cdot 5$

$30 = NO \cdot 5$ Divide both sides of the equation by 5

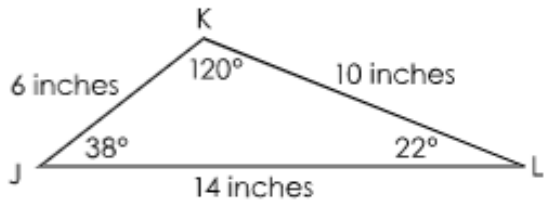
$NO = \frac{30}{5}$

NO = $\frac{30}{5}$ inches is a correct answer for the first question.

Since the original and the scaled (dilated) triangle are similar, the corresponding angles of the triangles are congruent, and their measures are equal. So, the angles L and O are corresponding and therefore, ∠O ≅ ∠L and m∠O = m∠L. So, the m ∠O = 22.0°.

**Sample Response: 1 point**

Megan dilates $\Delta JKL$ about point J by a scale factor of $\frac{3}{5}$ to create $\Delta JNO$.




What is the length, in inches, of $\overline{NO}$ ?

$NO =$ 6 *inches*

1 2 3
4 5 6
7 8 9
0
. -

What is the measure, in degrees, of $\angle O$ ?

13.2 *degrees*

1 2 3
4 5 6
7 8 9
0
. -

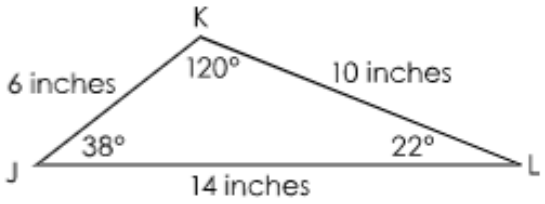
**Notes on Scoring**

This response earns partial credit (1 point) because it shows a correct length of side NO but an incorrect measure of ∠O.

The student may correctly calculate the length of side $\overline{\mathrm{NO}}$ to have a length of 6 inches but does not realize that if the original and the scaled (dilated) triangle are similar, their corresponding angles are congruent, and their measures are equal. Instead, to get the measure of angle O, the student may multiply the measure of angle L by the scale factor $\frac{3}{5}$, or $22 \cdot \frac{3}{5} = 13.2$.

**Sample Response: 1 point**

Megan dilates $\Delta JKL$ about point J by a scale factor of $\frac{3}{5}$ to create $\Delta JNO$.




What is the length, in inches, of $\overline{NO}$ ?

$NO =$ 10 *inches*

1 2 3
4 5 6
7 8 9
0
. -

What is the measure, in degrees, of $\angle O$ ?

22 *degrees*

1 2 3
4 5 6
7 8 9
0
. -

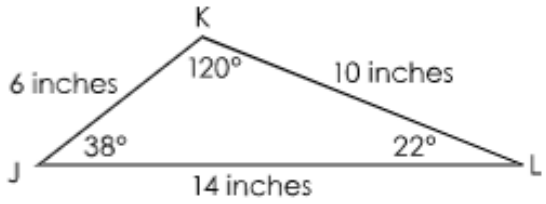
**Notes on Scoring**

This response earns a partial credit (1 point) because it shows a correct measure of ∠O but an incorrect length of side NO.

The student may incorrectly think that the scaled triangle is congruent to the original triangle so that their corresponding sides are congruent to conclude NO = KL = 10 in. The student correctly determines the measure of ∠O is 22°.

**Sample Response: 0 points**

Megan dilates $\Delta$ JKL about point J by a scale factor of $\frac{3}{5}$ to create $\Delta$ JNO.




What is the length, in inches, of $\overline{NO}$ ?

$NO =$ 10 *inches*

1 2 3
4 5 6
7 8 9
0
. -

What is the measure, in degrees, of $\angle O$ ?

13.2 *degrees*

1 2 3
4 5 6
7 8 9
0
. -

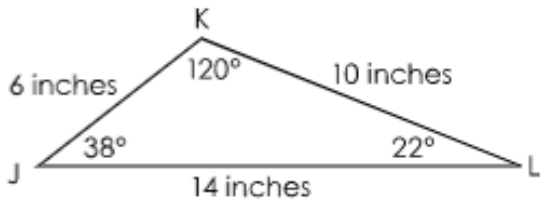
**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length of side NO and an incorrect measure of ∠O.

The student may realize that triangles JKL and JNO are similar but incorrectly applies the scale factor to the angle measures instead of the side lengths, and concludes that NO = KL = 10 in., and the measure of angle L = $22 \cdot \frac{3}{5} = 13.2°$.

**Sample Response: 0 points**

Megan dilates $\Delta JKL$ about point J by a scale factor of $\frac{3}{5}$ to create $\Delta JNO$.




What is the length, in inches, of $\overline{NO}$ ?

$NO =$ 8.4 *inches*

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | |

What is the measure, in degrees, of $\angle O$ ?

38 *degrees*

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | |

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length of side NO and an incorrect measure of ∠O.

The student finds the length of $\overline{\mathrm{JO}}$ instead of $\overline{\mathrm{NO}}$ and the measure of angle J instead of the measure of angle O.

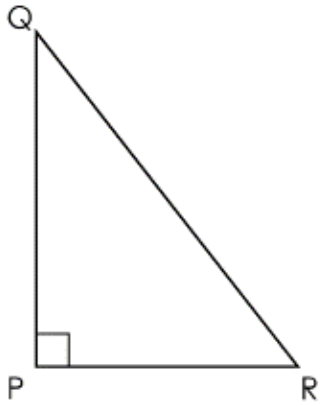
# Geometry
# Spring 2022 Item Release

# Question 34

# Question and Scoring Guidelines

# Question 34

Right triangle PQR is shown, where m∠Q is not equal to m∠R.




Which expression is equivalent to sin(Q)?

(A) sin(P)

(B) cos(P)

(C) sin(R)

(D) cos(R)

**Points Possible:** 1

**Content Cluster:** Define trigonometric ratios, and solve problems involving right triangles.

**Content Standard:** Explain and use the relationship between the sine and cosine of complementary angles. *(G.SRT.7)*

**Depth of Knowledge:** Level 1
a. Recall, observe, or recognize a fact, definition, term, or property
c. Apply a formula
n. Represent math relationships in words, pictures, or symbols

# Scoring Guidelines

Rationale for Option A: This is incorrect. The student may remember that the relationship involves a trigonometric ratio of another angle but confuses cosine with sine and the angle complementary to the angle Q with the right angle.

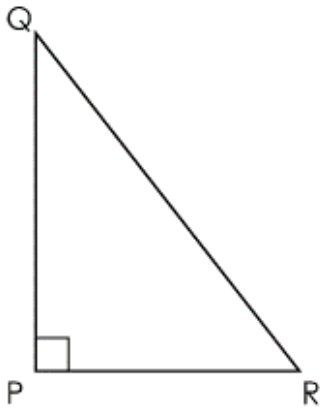
Rationale for Option B: This is incorrect. The student may remember that the relationship should involve a trigonometric ratio of another angle but confuses the angle complementary to the angle Q with the right angle.

Rationale for Option C: This is incorrect. The student may remember that the relationship involves a trigonometric ratio of the complementary angle but confuses cosine with sine of complementary angle R.

Rationale for Option D: **Key** – The student realizes that because sin Q = $\frac{PR}{QR}$ and cos R = $\frac{PR}{QR}$ , sin Q = cos R. So, the sine of the acute angle Q is equal to the cosine of its complement, angle R.

## Sample Response: 1 point

Right triangle PQR is shown, where m∠Q is not equal to m∠R.




Which expression is equivalent to sin(Q)?

- (A) sin(P)
- (B) cos(P)
- (C) sin(R)
- ● cos(R)

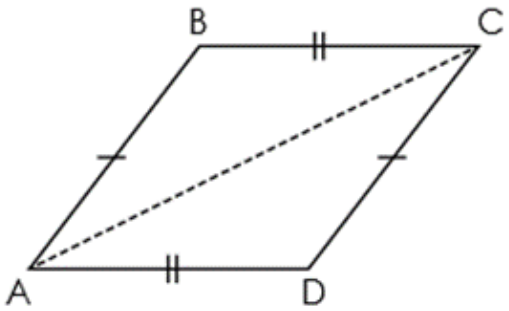
# Geometry
# Spring 2022 Item Release

# Question 37

# Question and Scoring Guidelines

## Question 37

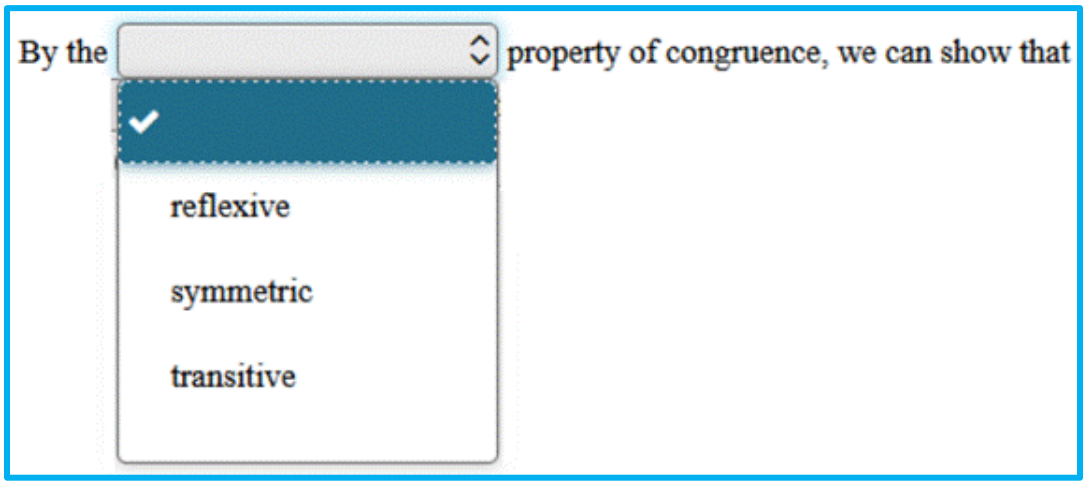
In quadrilateral ABCD, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, as shown.

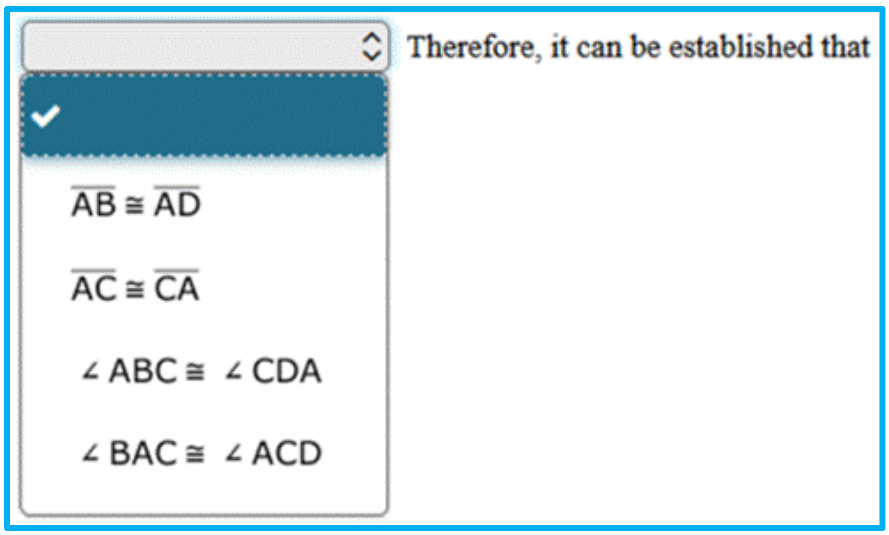


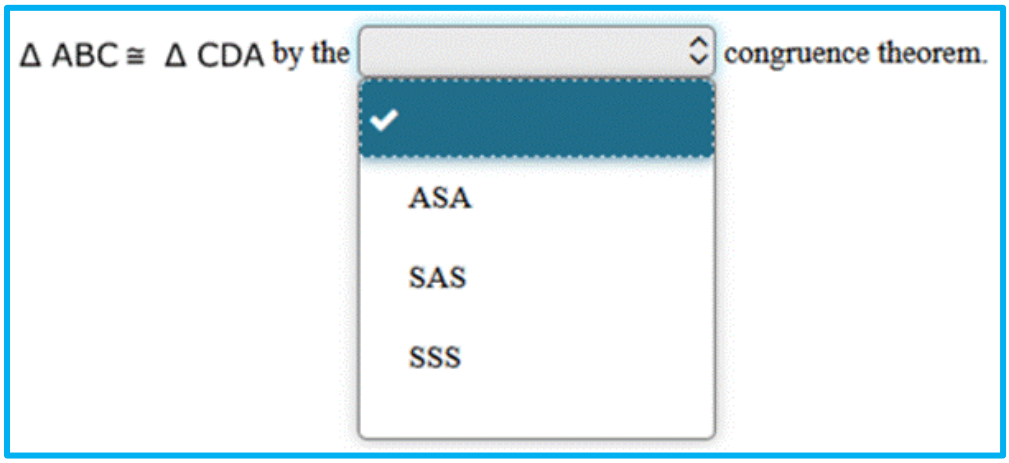

Select a term for each blank box to complete the proof showing that $\Delta ABC \cong \Delta CDA$.

By the [ ] property of congruence, we can show that [ ]. Therefore, it can be established that $\Delta ABC \cong \Delta CDA$ by the [ ] congruence theorem.

**Drop Down Choices:**



**Points Possible:** 1

**Content Cluster:** Prove and apply theorems both formally and informally involving similarity using a variety of methods.

**Content Standard:** Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles. *(G.SRT.5)*

**Depth of Knowledge:** Level 3
b. Explain thinking when more than one response is possible
d. Use evidence to develop logical arguments for a concept
e. Use concepts to solve non-routine problems
j. Provide mathematical justifications

# Scoring Guidelines

Exemplar Response

- By the **reflexive** property of congruence, we can show that $\overline{\mathbf{AC}} \cong \overline{\mathbf{CA}}$. Therefore, it can be established that $\Delta\,\mathrm{ABC} \cong \Delta\,\mathrm{CDA}$ by the **SSS** congruence theorem.

Other Correct Responses

- N/A

For full credit (1 point), the student's response satisfies the bullet below.

- The student correctly completes the proof.

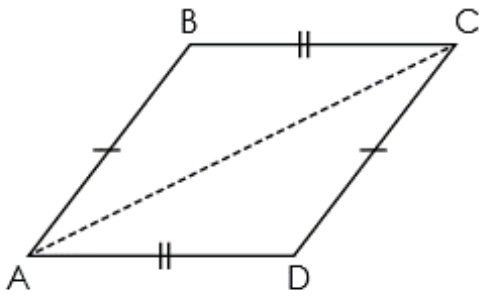
# Geometry
# Spring 2022 Item Release

# Question 37

# Sample Responses

## Sample Response: 1 point

In quadrilateral ABCD, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, as shown.




Select a term for each blank box to complete the proof showing that $\Delta ABC \cong \Delta CDA$.

By the [reflexive] property of congruence, we can show that [$\overline{AC} \cong \overline{CA}$]. Therefore, it can be established that $\Delta ABC \cong \Delta CDA$ by the [SSS] congruence theorem.

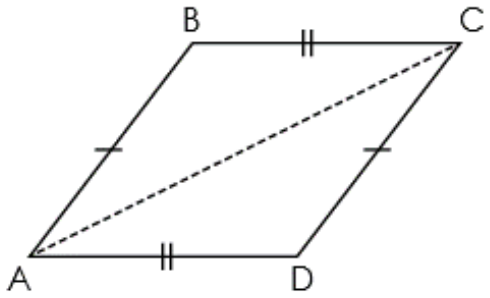
**Notes on Scoring**

This response earns full credit (1 point) because it shows three correctly selected terms to complete the proof.

To prove two triangles are congruent using the theorems for congruence of triangles, note that two pairs of sides are given to be congruent. It is reasonable to identify either another pair of congruent sides to claim a Side-Side-Side (SSS) triangles congruence theorem or a pair of congruent included angles to claim a Side-Angle-Side (SAS) triangles congruence theorem. Since ΔABC and ΔCDA share side AC, it can be used as the third pair of congruent sides, or AC ≅ CA. Therefore, the first correctly completed sentence is, "By the reflexive property of congruence, we can show that AC ≅ CA." Since AB ≅ CD and BC ≅ AD are given, and AC ≅ CA by the reflexive property, then ΔABC ≅ ΔCDA by SSS. So, the second correctly completed sentence is, "Therefore, it can be established that ΔABC ≅ ΔCDA by the SSS congruence theorem."

## Sample Response: 0 points

In quadrilateral ABCD, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, as shown.




Select a term for each blank box to complete the proof showing that $\Delta ABC \cong \Delta CDA$.

By the reflexive property of congruence, we can show that $\angle ABC \cong \angle CDA$. Therefore, it can be established that $\Delta ABC \cong \Delta CDA$ by the SAS congruence theorem.

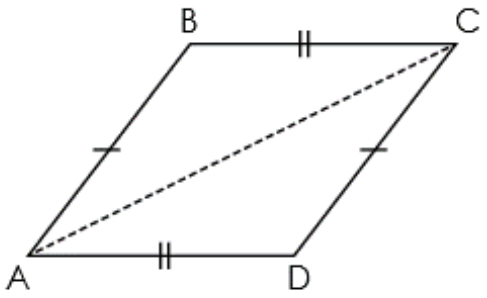
**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect proof to show $\Delta ABC \cong \Delta CDA$.

The student may confuse the reflexive property with a reflection and incorrectly think that a reflection across the dotted line will map angle B onto angle D. If it would have, then the congruence theorem would have been SAS.

## Sample Response: 0 points

In quadrilateral ABCD, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, as shown.




Select a term for each blank box to complete the proof showing that $\Delta ABC \cong \Delta CDA$.

By the [symmetric] property of congruence, we can show that [$\angle ABC \cong \angle CDA$]. Therefore, it can be established that $\Delta ABC \cong \Delta CDA$ by the [SAS] congruence theorem.

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect proof to show $\Delta ABC \cong \Delta CDA$.

The student may not realize that the symmetric property can only be used if we already know that two triangles are congruent. Having chosen angle B congruent to angle D, SAS would have been the correct congruence theorem.

# Geometry
# Spring 2022 Item Release

# Question 38

# Question and Scoring Guidelines

# Question 38

Events $Q$ and $R$ are independent events, such that $\frac{P(Q \cap R)}{P(R)} = \frac{4}{5}$.

Which probability is equal to $\frac{4}{5}$ ?

(A) $P(Q)$

(B) $P(R)$

(C) $P(Q \cup R)$

(D) $P(Q \cap R)$

**Points Possible:** 1

**Content Cluster:** Understand independence and conditional probability, and use them to interpret data.

**Content Standard:** Understand the conditional probability of A given B as $\frac{P(A \text{ and } B)}{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★ *(S.CP.3)*

**Depth of Knowledge:** Level 1
a. Recall, observe, or recognize a fact, definition, term, or property
c. Apply a formula
n. Represent math relationships in words, pictures, or symbols

# Scoring Guidelines

Rationale for Option A: **Key** – The student understands that for independent events $P(Q \cap R)/P(R) = P(R) \cdot P(Q)/P(R) = P(Q)$. Therefore, if $P(Q \cap R)/P(R) = 4/5$, then $P(Q) = 4/5$ as well.

Rationale for Option B: This is incorrect. The student may assume that for event Q and event R to be independent, $P(Q \cap R)/P(R) = P(R)$. Thus, the student selects $P(R) = 4/5$.

Rationale for Option C: This is incorrect. The student may assume that for event Q and event R to be independent, $P(Q \cap R)/P(R) = P(Q \cup R)$. Thus, the student selects $P(Q \cup R) = 4/5$.

Rationale for Option D: This is incorrect. The student may assume that for event Q and event R to be independent, $P(Q \cap R)/P(R) = P(Q \cap R)$. Thus, the student selects $P(Q \cap R) = 4/5$.

## Sample Response: 1 point

Events $Q$ and $R$ are independent events, such that $\frac{P(Q \cap R)}{P(R)} = \frac{4}{5}$.

Which probability is equal to $\frac{4}{5}$ ?

- ● $P(Q)$
- Ⓑ $P(R)$
- Ⓒ $P(Q \cup R)$
- Ⓓ $P(Q \cap R)$

# Geometry
# Spring 2022 Item Release

# Question 42

# Question and Scoring Guidelines

## Question 42

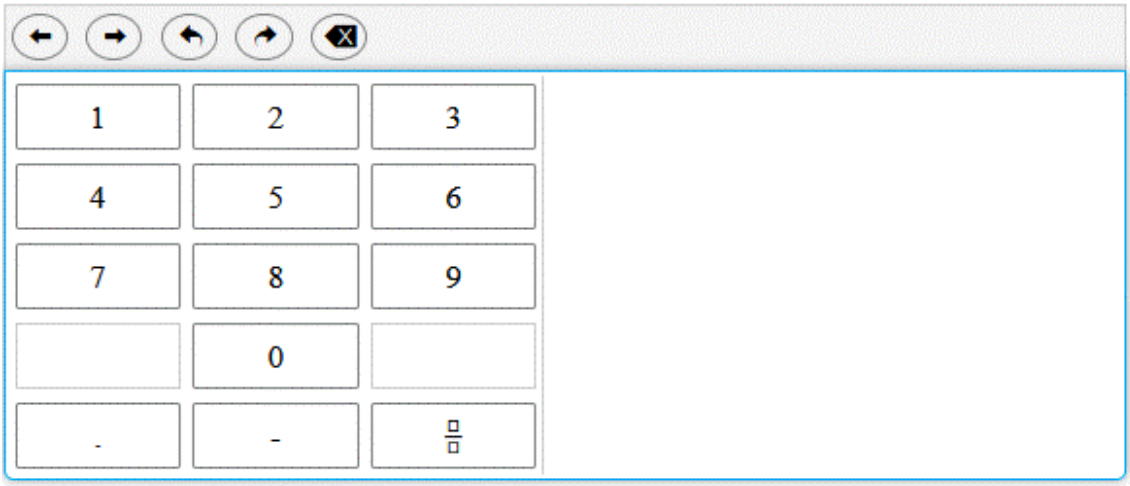
Gus is designing a cylinder to ship liquids using the constraints given.

- The inside of the cylinder must hold from 475 to 480 cubic centimeters of liquid.
- The diameter must be at least 8 centimeters and at most 10 centimeters.

What are a possible radius and corresponding height, in centimeters, for the inside of a cylinder that meets the constraints? Round the answers to the nearest tenth.

*Radius*: centimeters

*Height*: centimeters



**Points Possible:** 1

**Content Cluster:** Apply geometric concepts in modeling situations.

**Content Standard:** Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★ *(G.MG.3)*

**Depth of Knowledge:** Level 2
d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts
k. Make direct translations between problem situations and symbolic notation
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

- Radius: 4 centimeters
- Height: 9.50 centimeters

Other Correct Responses

- any radius *r* between 4 and 5 centimeters, inclusive, and a height which is greater than or equal to the minimum of $\frac{475}{\frac{22}{7}r^2}$ and $\frac{475}{\frac{22}{7}r^2}$ rounded to the nearest tenth and which is less than or equal to $\frac{480}{3.14r^2}$ and $\frac{480}{3.14r^2}$ rounded to the nearest tenth

For full credit (1 point), the student's response satisfies the bullet below.

- The student enters a correct combination for the radius and height.

# Geometry
# Spring 2022 Item Release

# Question 42

# Sample Responses

## Sample Response: 1 point

Gus is designing a cylinder to ship liquids using the constraints given.

- The inside of the cylinder must hold from 475 to 480 cubic centimeters of liquid.
- The diameter must be at least 8 centimeters and at most 10 centimeters.

What are a possible radius and corresponding height, in centimeters, for the inside of a cylinder that meets the constraints? Round the answers to the nearest tenth.

*Radius*: 4 *centimeters*

*Height*: 9.50 *centimeters*

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correctly identified possible radius and its corresponding height for the inside of the cylindrical object that is consistent with the given dimensions.

To design a cylinder with the diameter that is at least 8 cm and at most 10 cm and the volume that is not larger than 480 cubic centimeters but not smaller than 475 cubic centimeters, Gus needs to determine the largest and the smallest radius of the base. The minimum and maximum values for the radius can be found by dividing the diameter by 2. So, the smallest radius for the inside of the cylinder is 4 cm and the largest radius is 5 cm.

The corresponding height values can be found by using the formula for the volume of the cylinder, $V = \pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cylinder. After solving this formula for the height, $h = \frac{V}{\pi r^2}$. The value of h is the smallest when the value of $V$ is the smallest, or 475, and the value of r is the largest, or 5. Therefore, $h = \frac{475}{\pi 5^2}$ or $h \approx 6.05$ cm.

The value of $h$ is the largest when the value of $V$ is the largest, or 480, and the value of $r$ is the smallest, or 4. Therefore, $h = \frac{480}{\pi 4^2}$ or $h \approx 9.50$ cm.

## Sample Response: 1 point

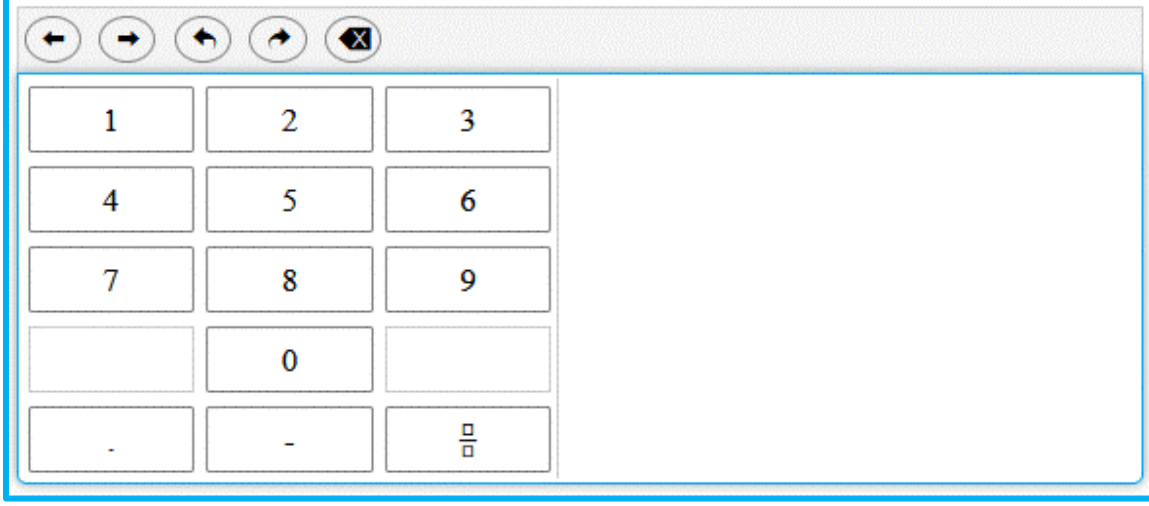
Gus is designing a cylinder to ship liquids using the constraints given.

- The inside of the cylinder must hold from 475 to 480 cubic centimeters of liquid.
- The diameter must be at least 8 centimeters and at most 10 centimeters.

What are a possible radius and corresponding height, in centimeters, for the inside of a cylinder that meets the constraints? Round the answers to the nearest tenth.

*Radius*: 5 *centimeters*

*Height*: 6.11 *centimeters*



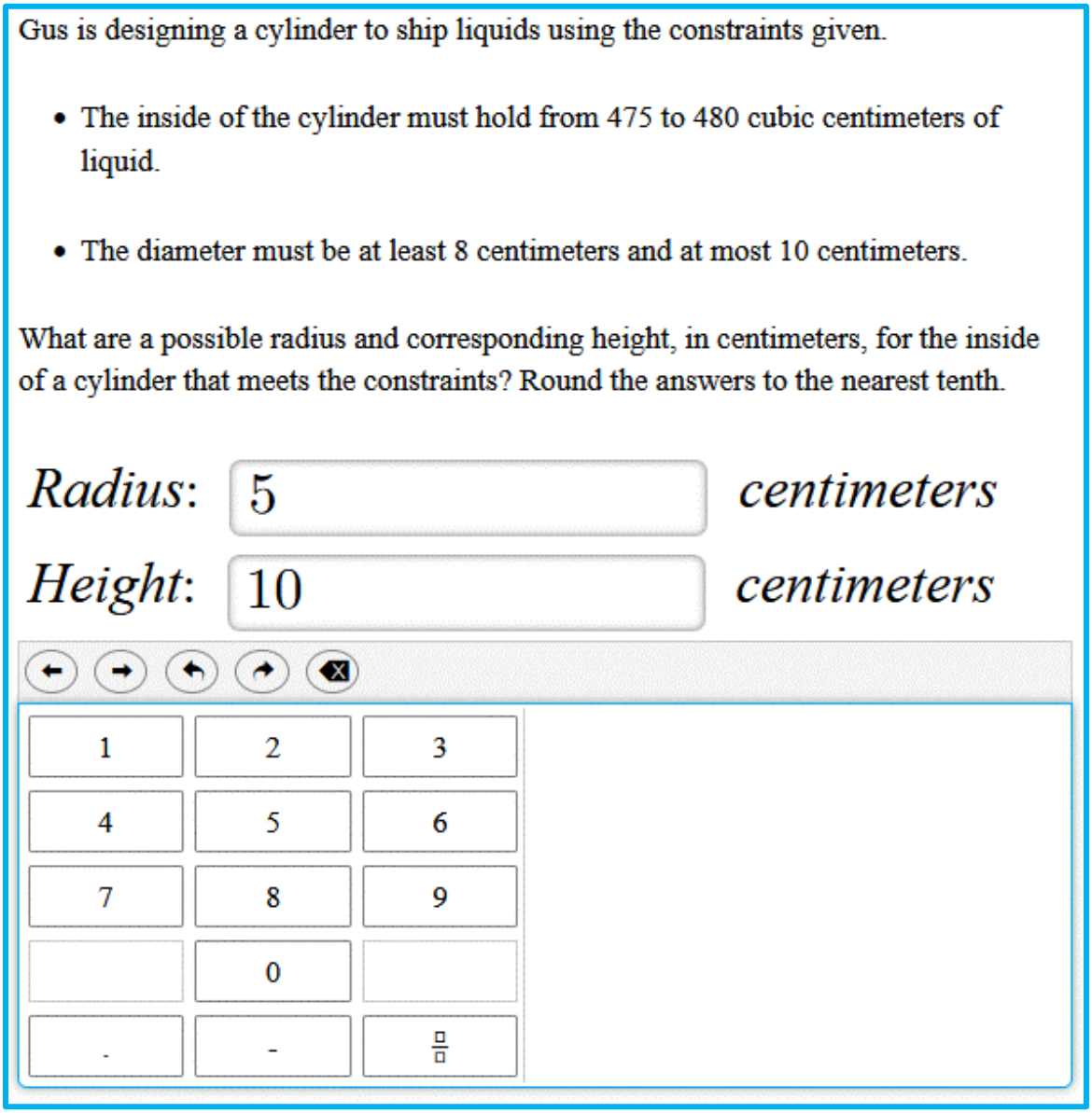
**Notes on Scoring**

This response earns full credit (1 point) because it shows a correctly identified possible radius and its corresponding height for the inside of the cylindrical object that is consistent with the given dimensions.

To determine the radius and the height, the student may decide to use the largest diameter of 10 cm and the largest volume of 480 $cm^3$.

First, the student divides the diameter by 2 to get the radius of 5 cm. Then, the student finds the corresponding height by using the formula for the volume of the cylinder, $V = \pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cylinder, after solving this formula for the height, $h = \frac{V}{\pi r^2}$. Next, the student evaluates this formula for $V = 480$ and $r = 5$, to calculate $h = \frac{480}{\pi 5^2} \approx 6.11$ cm.

## Sample Response: 0 points




**Notes on Scoring**

This response earns no credit (0 points) because it shows a correct possible radius but incorrect corresponding height for the inside of the cylindrical object.

The student correctly identifies a possible radius of 5 centimeters but may confuse the height of the cylinder with the diameter.

## Sample Response: 0 points

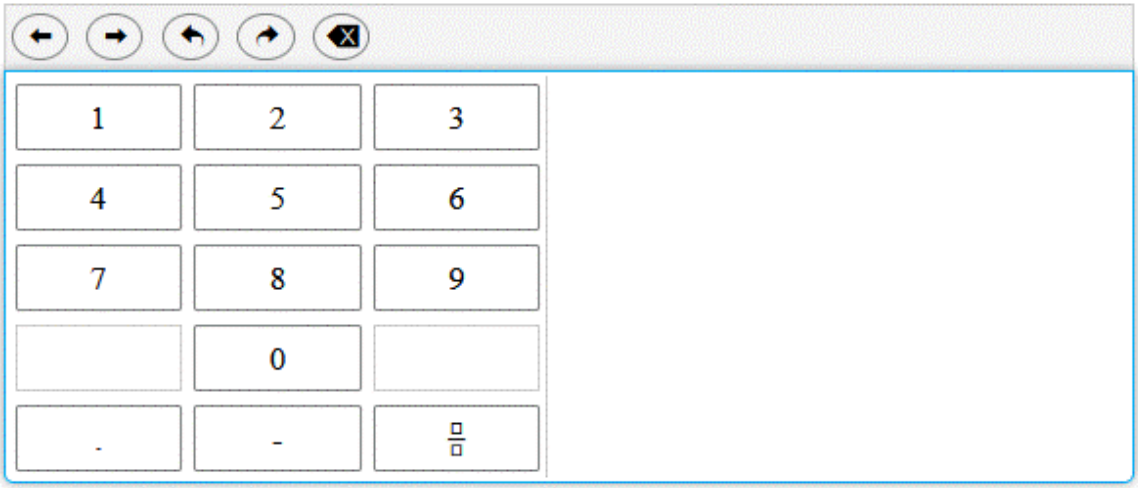
Gus is designing a cylinder to ship liquids using the constraints given.

- The inside of the cylinder must hold from 475 to 480 cubic centimeters of liquid.
- The diameter must be at least 8 centimeters and at most 10 centimeters.

What are a possible radius and corresponding height, in centimeters, for the inside of a cylinder that meets the constraints? Round the answers to the nearest tenth.

*Radius*: 16 *centimeters*

*Height*: 0.59 *centimeters*



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect possible radius and incorrect corresponding height for the inside of the cylindrical object.

The student may multiply a possible diameter of 8 by 2 instead of dividing by 2 to obtain the radius. The student then calculates the height of the cylinder based on the incorrect radius as $h = \frac{475}{\pi 16^2} \approx 0.59$ cm.

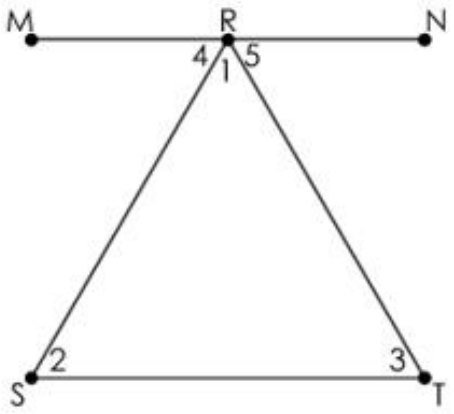
# Geometry
# Spring 2022 Item Release

# Question 43

# Question and Scoring Guidelines

# Question 43

In the figure shown, $\overline{MN} \parallel \overline{ST}$.




Move statements and reasons into the blank boxes to complete the proof that $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

| Statements | Reasons |
|---|---|
| 1. $\overline{MN} \parallel \overline{ST}$ | 1. Given |
| 2. $m\angle 2 = m\angle 4$ and $m\angle 3 = m\angle 5$ | 2. |
| 3. $m\angle 4 + m\angle 1 = m\angle MRT$ | 3. Angle addition postulate |
| 4. | 4. |
| 5. $m\angle 1 + m\angle 4 + m\angle 5 = 180°$ | 5. Substitution |
| 6. | 6. Substitution |

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

$m\angle MRT + m\angle 5 = 180°$

$m\angle MRT + m\angle 2 = 180°$

Linear pair theorem

Definition of complementary angles

Vertical angles are congruent.

Alternate exterior angles are congruent.

Alternate interior angles are congruent.

**Points Possible:** 1

**Content Cluster:** Prove geometric theorems both formally and informally using a variety of methods.

**Content Standard:** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)*

**Depth of Knowledge:** Level 3
b. Explain thinking when more than one response is possible
d. Use evidence to develop logical arguments for a concept
e. Use concepts to solve non-routine problems
j. Provide mathematical justifications

# Scoring Guidelines

Exemplar Response

| Statements | Reasons |
|---|---|
| 1. $\overline{MN} \parallel \overline{ST}$ | 1. Given |
| 2. $m\angle 2 = m\angle 4$ and $m\angle 3 = m\angle 5$ | 2. Alternate interior angles are congruent. |
| 3. $m\angle 4 + m\angle 1 = m\angle MRT$ | 3. Angle addition postulate |
| 4. $m\angle MRT + m\angle 5 = 180°$ | 4. Linear pair theorem |
| 5. $m\angle 1 + m\angle 4 + m\angle 5 = 180°$ | 5. Substitution |
| 6. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ | 6. Substitution |

Other Correct Responses

- N/A

For this item, a full-credit response includes:

- a correctly completed proof (1 point).

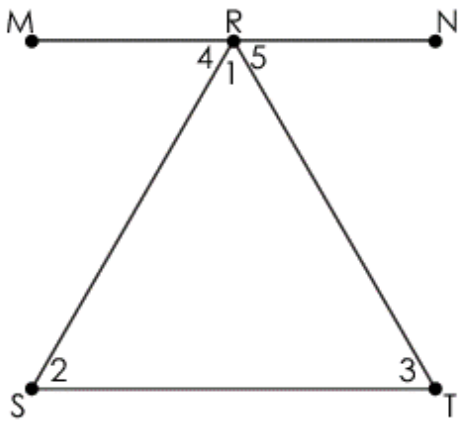
# Geometry
# Spring 2022 Item Release

# Question 43

# Sample Responses

## Sample Response: 1 point

In the figure shown, $\overline{MN} \parallel \overline{ST}$.




Move statements and reasons into the blank boxes to complete the proof that $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

| Statements | Reasons |
|---|---|
| 1. $\overline{MN} \parallel \overline{ST}$ | 1. Given |
| 2. $m\angle 2 = m\angle 4$ and $m\angle 3 = m\angle 5$ | 2. Alternate interior angles are congruent. |
| 3. $m\angle 4 + m\angle 1 = m\angle MRT$ | 3. Angle addition postulate |
| 4. $m\angle MRT + m\angle 5 = 180°$ | 4. Linear pair theorem |
| 5. $m\angle 1 + m\angle 4 + m\angle 5 = 180°$ | 5. Substitution |
| 6. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ | 6. Substitution |

**Notes on Scoring**

This response earns full credit (1 point) because it correctly completes the proof to show that the sum of three interior angles of the triangle is 180° (Triangle Sum Theorem).

To complete a mathematics proof, a correctly chosen piece of missing information should show where each statement is logically connected with the previous statement, which then allows for the next correct statement.

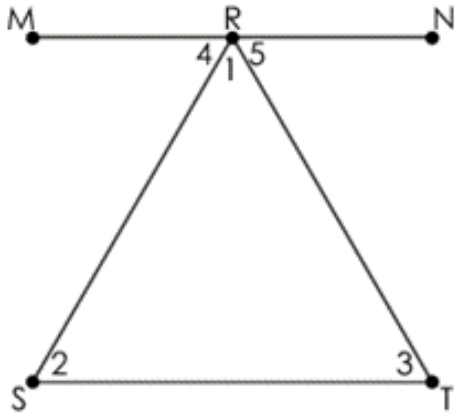
Since line segments MN and ST are given to be parallel and SR is a transversal, $\angle 2 \cong \angle 4$. Likewise, since line segments MN and ST are given to be parallel and TR is a transversal, $\angle 3 \cong \angle 5$. The reason for these two conclusions is the theorem stating that alternate interior angles are congruent, if angles are formed by two parallel lines and a transversal. So, Reason 2 is "Alternate interior angles are congruent."

Statement 3 shows m $\angle 4$ + m $\angle 1$ = m $\angle MRT$. Statement 5 shows that m $\angle MRT$ substitutes m $\angle 4$ + m $\angle 1$. Therefore, Statement 4 should logically connect to Statements 3 and 5. According to the diagram, $\angle MRT$ and $\angle 5$ form a linear pair whose measure is 180°. So, Statement 4 is m $\angle MRT$ + m $\angle 1$ = 180 and Reason 4 is Linear pair theorem.

Statement 6, m $\angle 1$ + m $\angle 2$ + m $\angle 3$ = 180, and the Reason 6, "Substitution," completes the proof, where m $\angle 2$ substitutes m $\angle 4$ and m $\angle 3$ substitutes m $\angle 5$.

## Sample Response: 0 points

In the figure shown, $\overline{MN} \parallel \overline{ST}$.




Move statements and reasons into the blank boxes to complete the proof that $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

| Statements | Reasons |
|---|---|
| 1. $\overline{MN} \parallel \overline{ST}$ | 1. Given |
| 2. $m\angle 2 = m\angle 4$ and $m\angle 3 = m\angle 5$ | 2. Alternate exterior angles are congruent. |
| 3. $m\angle 4 + m\angle 1 = m\angle MRT$ | 3. Angle addition postulate |
| 4. $m\angle MRT + m\angle 5 = 180°$ | 4. Definition of complementary angles |
| 5. $m\angle 1 + m\angle 4 + m\angle 5 = 180°$ | 5. Substitution |
| 6. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ | 6. Substitution |

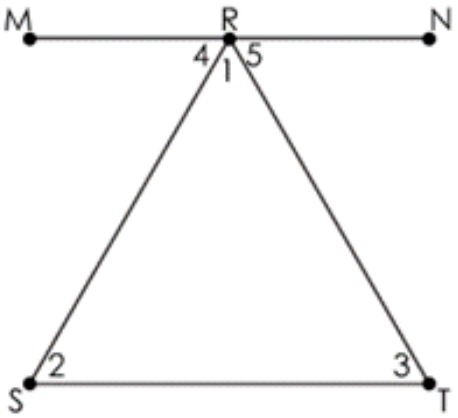
**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect proof.

In statement 2, for two pairs of congruent alternate interior angles the student selects an incorrect reason "Alternate exterior angles are congruent." In statement 4, the student confuses a pair supplementary angles, whose sum is 180 degrees, with a pair of complementary angles, whose sum is 90 degrees.

## Sample Response: 0 points

In the figure shown, $\overline{MN} \parallel \overline{ST}$.




Move statements and reasons into the blank boxes to complete the proof that $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

| Statements | Reasons |
|---|---|
| 1. $\overline{MN} \parallel \overline{ST}$ | 1. Given |
| 2. $m\angle 2 = m\angle 4$ and $m\angle 3 = m\angle 5$ | 2. Alternate interior angles are congruent. |
| 3. $m\angle 4 + m\angle 1 = m\angle MRT$ | 3. Angle addition postulate |
| 4. $m\angle MRT + m\angle 2 = 180°$ | 4. Definition of complementary angles |
| 5. $m\angle 1 + m\angle 4 + m\angle 5 = 180°$ | 5. Substitution |
| 6. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ | 6. Substitution |

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect proof. In statement 4, the statement and the reason are incorrect.

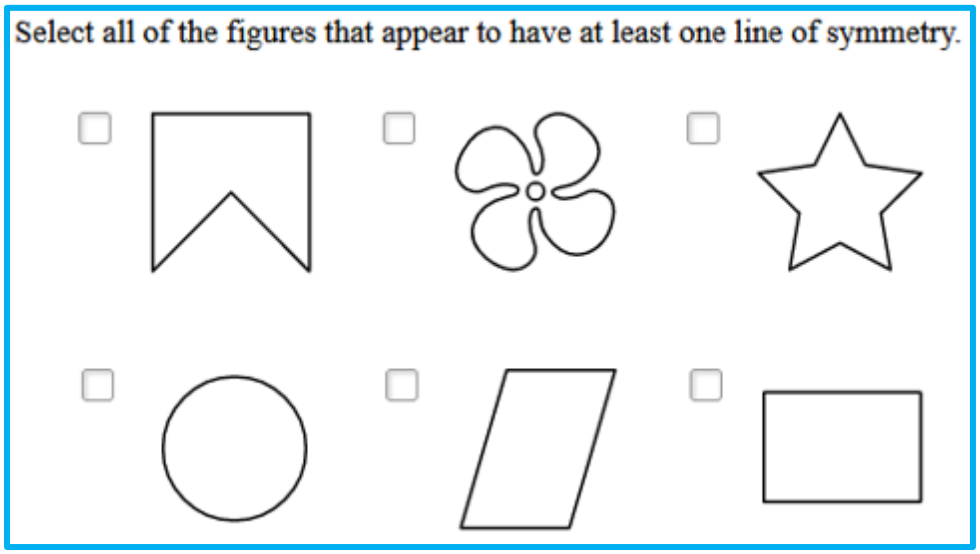
The student selects the measure of angle 2 instead of the measure of angle 5 and selects a Definition of complementary angles, whose sum is 90 degrees instead of a definition of Supplementary angles whose sum is 180 degrees.

# Geometry
# Spring 2022 Item Release

# Question 45

# Question and Scoring Guidelines

# Question 45




**Points Possible:** 1

**Content Cluster:** Experiment with transformations in the plane.

**Content Standard:** Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself.
*a.* Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. *(G.CO.3)*

**Depth of Knowledge:** Level 1
a. Recall, observe, or recognize a fact, definition, term, or property
j. Retrieve information from a table or graph
n. Represent math relationships in words, pictures, or symbols

# Scoring Guidelines

Rationale for First Option: **Key** – The student correctly identifies that there is one vertical line of symmetry that goes through the midpoint of the horizontal side and the middle vertex of the figure. A reflection across this line will map the figure onto itself.

Rationale for Second Option: This is incorrect. The student may correctly note that this figure has rotational symmetry, since rotating it by 90 degrees in either direction about the center would map the figure onto itself, but misses that this figure does not have any lines of symmetry.

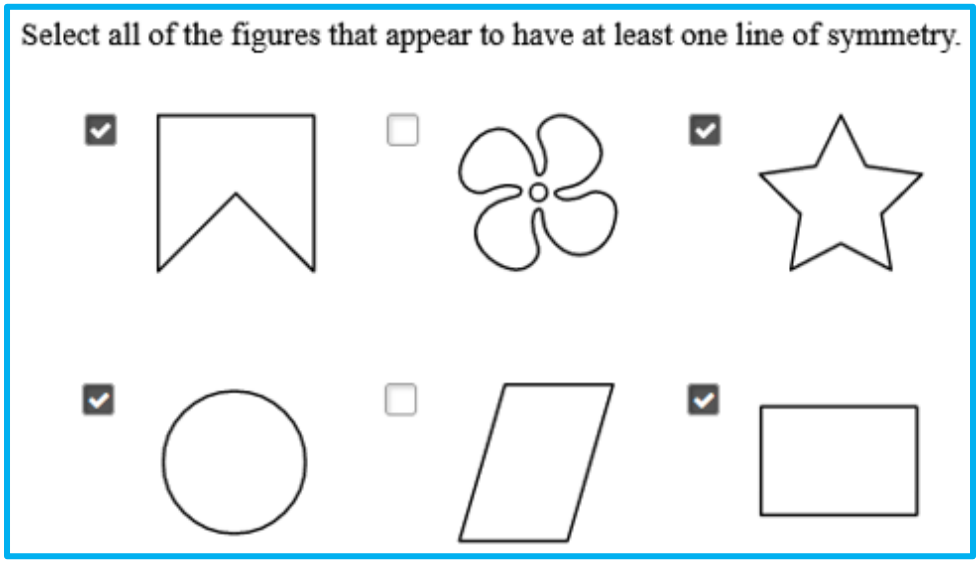
Rationale for Third Option: **Key** – The student correctly identifies that this figure has multiple lines of symmetry bisecting the points of the star.

Rationale for Fourth Option: **Key** – The student correctly notes that all lines through the center of the circle are lines of symmetry for the circle.

Rationale for Fifth Option: This is incorrect. The student may incorrectly think that diagonals of the parallelogram are the lines of symmetry. A reflection across those line will not map the figure onto itself.

Rationale for Sixth Option: **Key** – The student correctly identifies that this figure has two lines of symmetry, one is a vertical segment bisector of the horizontal sides and the other is a horizontal segment bisector of the vertical sides.

## Sample Response: 1 point



# Geometry
# Spring 2022 Item Release

# Question 48

# Question and Scoring Guidelines

# Question 48

Janet draws triangle PQR with vertices P (1, 1), Q (–2, 4), and R (1, 7). She claims that the triangle is an isosceles right triangle.

Which statement shows that Janet is correct?

Ⓐ The slope of $\overline{PQ}$ is 1, which is the reciprocal of the slope of $\overline{QR}$, and PQ and QR are equal to $\sqrt{6}$.

Ⓑ The slope of $\overline{PQ}$ is 1, which is the reciprocal of the slope of $\overline{QR}$, and PQ and QR are equal to $3\sqrt{2}$.

Ⓒ The slope of $\overline{PQ}$ is –1, which is the negative reciprocal of the slope of $\overline{QR}$, and PQ and QR are equal to $\sqrt{6}$.

Ⓓ The slope of $\overline{PQ}$ is –1, which is the negative reciprocal of the slope of $\overline{QR}$, and PQ and QR are equal to $3\sqrt{2}$.

**Points Possible:** 1

**Content Cluster:** Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

**Content Standard:** Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle. (G.GPE.4)*

**Depth of Knowledge:** Level 2
c. Use models to represent mathematical concepts
d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts
e. Compare and/or contrast figures or statements

# Scoring Guidelines

Rationale for Option A: This is incorrect. The student may incorrectly calculate the slope of $\overline{PQ}$ as $\frac{4-1}{1-(-2)} = \frac{3}{3} = 1$ and incorrectly believe that lines with reciprocal slopes are perpendicular. The student incorrectly calculates the side lengths as $PQ = \sqrt{1-(-2)+(4-1)} = \sqrt{3+3} = \sqrt{6}$ and $QR = \sqrt{1-(-2)+(7-4)} = \sqrt{3+3} = \sqrt{6}$.

Rationale for Option B: This is incorrect. The student may incorrectly calculate the slope of $\overline{PQ}$ as $\frac{4-1}{1-(-2)} = \frac{3}{3} = 1$ and incorrectly believe that lines with reciprocal slopes are perpendicular. However, the student correctly calculates the side lengths as $PQ = \sqrt{(1-(-2))^2+(1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$ and $QR = \sqrt{(-2-1)^2+(4-7)^2} = \sqrt{9+9} = 3\sqrt{2}$.

Rationale for Option C: This is incorrect. The student may incorrectly calculate the side lengths as $PQ = \sqrt{1-(-2)+(4-1)} = \sqrt{3+3} = \sqrt{6}$ and $QR = \sqrt{1-(-2)+(7-4)} = \sqrt{3+3} = \sqrt{6}$. However, the student correctly calculates the slope of $\overline{PQ}$ as $\frac{4-1}{-2-1} = \frac{3}{-3} = -1$ and the slope of $\overline{QR}$ as $\frac{7-4}{1-(-2)} = \frac{3}{3} = 1$.

Rationale for Option D: **Key** – The student correctly calculates the slope of $\overline{PQ}$ as $\frac{4-1}{-2-1} = \frac{3}{-3} = -1$ and the slope of $\overline{QR}$ as $\frac{7-4}{1-(-2)} = \frac{3}{3} = 1$, correctly calculates the side lengths $PQ = \sqrt{(1-(-2))^2+(1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$ and $QR = \sqrt{(-2-1)^2+(4-7)^2} = \sqrt{9+9} = 3\sqrt{2}$, to deduce that the triangle is an isosceles right triangle because the side lengths are equal and the slopes are opposite reciprocals.

## Sample Response: 1 point

Janet draws triangle PQR with vertices P (1, 1), Q (–2, 4), and R (1, 7). She claims that the triangle is an isosceles right triangle.

Which statement shows that Janet is correct?

Ⓐ The slope of $\overline{PQ}$ is 1, which is the reciprocal of the slope of $\overline{QR}$, and PQ and QR are equal to $\sqrt{6}$.

Ⓑ The slope of $\overline{PQ}$ is 1, which is the reciprocal of the slope of $\overline{QR}$, and PQ and QR are equal to $3\sqrt{2}$.

Ⓒ The slope of $\overline{PQ}$ is –1, which is the negative reciprocal of the slope of $\overline{QR}$, and PQ and QR are equal to $\sqrt{6}$.

● The slope of $\overline{PQ}$ is –1, which is the negative reciprocal of the slope of $\overline{QR}$, and PQ and QR are equal to $3\sqrt{2}$.